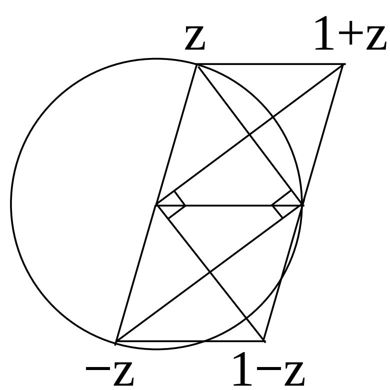
Question
By the means of the Argand diagram,or otherwise, show that if $|z| = 1$ the real part of $\dfrac{1+z}{1-z}$ is zero.

Answer

$$\begin{aligned}\frac{1+e^{i\theta}}{1-e^{i\theta}} &= \frac{e^{-i\frac{\theta}{2}}+e^{i\frac{\theta}{2}}}{e^{-i\frac{\theta}{2}}-e^{i\frac{\theta}{2}}} \\ &= \frac{2\cos\frac{\theta}{2}}{-2i\sin\frac{\theta}{2}} \\ &= i\cot\frac{\theta}{2}\end{aligned}$$



Therefore $\arg(1+z) - \arg(1-z) = \frac{\pi}{2}$ ($-\frac{\pi}{2}$ if z is replaced by -z) In either case the ra=eal part is zero.