QUESTION

Find the vector equation of the plane Π_1 which passes through the points $L = (1, 1, 0)$, $M = (1, -2, 2)$ and $N = (3, 0, 3)$. What is the equation of the plane in terms of x, y, z coordinates?

A second plane Π_2 is parallel to Π_1 and passes through the point $Q = (1, 1, 1)$. Find the equation of Π_2 in terms of x, y, z coordinates.

Give the parametric equation for the line ℓ through the point Q orthogonal to Π_1, and find the point A where it intersects the plane Π_1. Write down the vector joining the point L to A, and verify that this is orthogonal to the line ℓ.

ANSWER

$$\mathbf{u} = \vec{LM} = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}, \ \mathbf{v} = \vec{LN} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & 2 \\ 2 & -1 & 3 \end{vmatrix} = -7\mathbf{i} + 4\mathbf{j} + 6\mathbf{k} = \begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix}$$

so the equation of Π_1 is $\begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix}.\mathbf{w} = \begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = -3$

In co-ordinates this is $-7x + 4y + 6z = -3$.

Π_2 has equation $-7x + 4y + 6z = 3$. ℓ has equation $(1, 1, 1) + t(-7, 4, 6)$ or $(x, y, z) = (1 - 7t, 1 + 4t, 1 + 6t)$ This point lies in $\Pi_1 \Leftrightarrow -7(1 - 7t) + 4(1 + 4t) + 6(1 + 6t) = -3 \Leftrightarrow -7 + 4 + 6 + 3 = (-49 - 16 - 36)t$ i.e. $t = \frac{-6}{101}$ hence $A = \left(\frac{143}{101}, \frac{77}{101}, \frac{65}{101}\right)$.

$\vec{LA}$ is orthogonal to $\ell \Leftrightarrow \vec{LA}.(\mathbf{u} \times \mathbf{v}) = 0$

$\vec{LA} = \left(\frac{42}{101}, -\frac{24}{101}, \frac{65}{101}\right)^T$, $\vec{LA}.\begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix} = -284 - 96 + 390 = 0.$