Question
Consider the following set of simultaneous equations

$$\begin{aligned} x+y+z &= 0, \\ 2x-3y-z &= 1, \\ 2y+z &= 2. \end{aligned}$$

Find the solution by matrix inversion.
Note: If you fail to show detailed working of the matrix inversion, no marks will be awarded, even if you can write down the correct answer.

Answer
$k=0$

$$\begin{aligned} x+y+z &= 0 \\ 2x-3y-z &= 1 \\ 2y+z &= 2 \end{aligned}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & -1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$
$\mathbf{A} \cdot \quad \mathbf{x} \quad = \quad \mathbf{b}$
Need $\mathbf{A}^{-1}$

$$\begin{aligned} detA & \\ &= 1\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix} \\ &= -1-2+4 \\ &= 1 \neq 0 \end{aligned}$$

so $\mathbf{A}^{-1}$ exists.
Form matrix of cofactors
cofactor of $A_{11} = +\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} = -1$ Following + - sign pattern
cofactor of $A_{12} = -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -2$
cofactor of $A_{13} = +\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix} = +4$
cofactor of $A_{21} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = +1$

cofactor of $A_{22} = + \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = +1$

cofactor of $A_{23} = - \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = -2$

cofactor of $A_{31} = + \begin{vmatrix} 1 & 1 \\ -3 & -1 \end{vmatrix} = +2$

cofactor of $A_{32} = - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = +3$

cofactor of $A_{33} = + \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -5$

Cofactor matrix $= \begin{pmatrix} -1 & -2 & 4 \\ 1 & 1 & -2 \\ 2 & 3 & 5 \end{pmatrix}$

Transpose $= \begin{pmatrix} -1 & 1 & 2 \\ -2 & 1 & 3 \\ 4 & -2 & -5 \end{pmatrix}$

$\frac{\text{Transpose}}{\text{det}} = \frac{1}{1} \begin{pmatrix} -1 & 1 & 2 \\ -2 & 1 & 3 \\ 4 & -2 & -5 \end{pmatrix}$

Therefore

$A^{-1} = \begin{pmatrix} -1 & 1 & 2 \\ -2 & 1 & 3 \\ 4 & -2 & -5 \end{pmatrix}$

Therefore

$$\begin{aligned} \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} -1 & 1 & 2 \\ -2 & 1 & 3 \\ 4 & -2 & -5 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 7 \\ -12 \end{pmatrix} \end{aligned}$$

Therefore $x = 5,\ y = 7,\ z = -12$