Question

Answer

(i) $2x + y - 2z = 3$

$\mathbf{r} \cdot \hat{\mathbf{n}} = d$

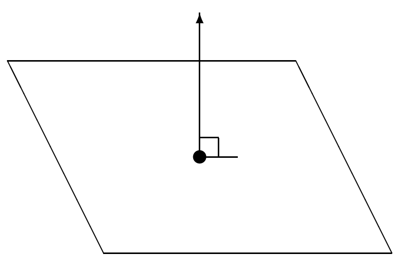
$\underbrace{(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})}_{\mathbf{n}} \cdot \underbrace{(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})}_{\mathbf{r}} = 3$

$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{2\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

Therefore $\mathbf{r} \cdot \hat{\mathbf{n}}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot \left(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \frac{2}{3} = |\hat{\mathbf{n}}|$

Therefore $\mathbf{r} \cdot \left(\underbrace{\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - ds\frac{2}{3}\mathbf{k}}_{\hat{\mathbf{n}}}\right) = 1$

$\hat{\mathbf{n}}$ is the unit normal vector to the plane



d is the perpendicular distance from orange $\underline{= 1}$.

(ii) **(a)** Intersection: Substitute $\mathbf{r} = -4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$

into $\mathbf{r} \cdot \hat{\mathbf{n}} = d$

Therefore $[(-4 + 2\lambda)\mathbf{i} + (2 + \lambda)\mathbf{j} + (1 + \lambda)\mathbf{k}] \cdot \left[\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right] = 1$

$\Rightarrow (-4 + 2\lambda)\frac{2}{3} + (2 + \lambda)\frac{1}{3} - (1 + \lambda)\frac{2}{3} = 1$

$\Rightarrow -8 + 4\lambda + 2 + \lambda - 2 - 2\lambda = 3$

$\Rightarrow 3\lambda = 11 \Rightarrow \underline{\lambda = \frac{11}{3}}$

Therefore break in line,

$$\begin{aligned} \mathbf{r} &= -4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \frac{11}{3}(2\mathbf{i} + \mathbf{j} + \mathbf{k}) \\ &= \frac{10}{3}\mathbf{i} + \frac{17}{3}\mathbf{j} + \frac{14}{3}\mathbf{k} \end{aligned}$$

(b) Substitute $\mathbf{r} = 3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{k})$

into $\mathbf{r} \cdot \hat{\mathbf{n}} = d$

$(3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \mu\mathbf{i} + \mu\mathbf{k}) \cdot \left(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = 1$

$(3 + \mu) \times \frac{2}{3} + 3 \times \frac{1}{3} + (2 + \mu) \times -\frac{2}{3} = 1$

$\Rightarrow 6 + 2\mu + 3 - 4 - 2\mu = 3$

$\Rightarrow \underline{5 = 3}$!!!

So no value of μ can be found.

Therefore no intersection.

Therefore line is parallel to plane: