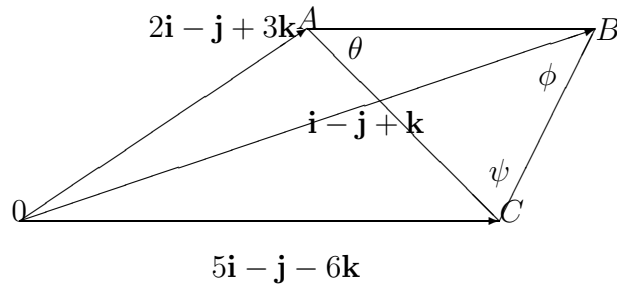


Question

Answer

(i)



(ii) $\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \mathbf{i} - \mathbf{j} + \mathbf{k} = -\mathbf{i} - 2\mathbf{k}$

$\mathbf{AC} = -\mathbf{OA} + \mathbf{OC} = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + 5\mathbf{i} - \mathbf{j} - 6\mathbf{k} = +3\mathbf{i} - 9\mathbf{k}$

$\mathbf{AB} \cdot \mathbf{AC} = |\mathbf{AB}||\mathbf{AC}| \cos \theta$

so $\cos \theta = \frac{\mathbf{AB} \cdot \mathbf{AC}}{|\mathbf{AB}||\mathbf{AC}|}$

$|\mathbf{AB}| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}$

$|\mathbf{AC}| = \sqrt{3^2 + 0^2 + 9^2} = \sqrt{90}$

Therefore

$$\begin{aligned} \cos \theta &= \frac{(-\mathbf{i} - 2\mathbf{k}) \cdot (3\mathbf{i} - 9\mathbf{k})}{\sqrt{5}\sqrt{90}} \\ &= \frac{(-1 \times 3 + 0 + 2 \times 9)}{\sqrt{5}\sqrt{90}} \\ &= \frac{15}{\sqrt{5}\sqrt{90}} \\ &= 0.707 \end{aligned}$$

$\Rightarrow \theta = \cos^{-1} \left(\frac{15}{\sqrt{5}\sqrt{90}} \right) = 45^\circ$

$\mathbf{BA} = -\mathbf{AB} = \mathbf{i} + 2\mathbf{k}$

$\mathbf{BC} = -\mathbf{OB} + \mathbf{OC} = -\mathbf{i} + \mathbf{j} - \mathbf{k} + 5\mathbf{i} - \mathbf{j} - 6\mathbf{k} = 4\mathbf{i} - 7\mathbf{k}$

$$\mathbf{BA} \cdot \mathbf{BC} = |\mathbf{BA}||\mathbf{BC}| \cos \phi$$

$$\text{so } \cos \phi = \frac{\mathbf{BA} \cdot \mathbf{BC}}{|\mathbf{BA}||\mathbf{BC}|}$$

$$|\mathbf{BA}| = |\mathbf{AB}| = \sqrt{5}$$

$$|\mathbf{BC}| = \sqrt{4^2 + 0^2 + 7^2} = \sqrt{65}$$

Therefore

$$\begin{aligned} \cos \phi &= \frac{(\mathbf{i} + 2\mathbf{k}) \cdot (4\mathbf{i} - 7\mathbf{k})}{\sqrt{5}\sqrt{65}} \\ &= \frac{(4 \times 1 + 0 + 7 \times 2)}{\sqrt{5}\sqrt{65}} \\ &= \frac{-10}{\sqrt{5}\sqrt{65}} \\ &= -0.5547 \end{aligned}$$

$$\Rightarrow \phi = \cos^{-1}(-0.5547) = 123.69^\circ$$

$$\text{Therefore } \psi = 180 - \theta - \phi = 180 - 45 - 123.69 = 11.3^\circ$$

(iii)

$$\begin{aligned} \text{Area of triangle ABC} &= \frac{1}{2} ||\mathbf{AB} \times \mathbf{AC}|| \text{ (e.g)} \\ &= \frac{1}{2} \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 0 & -2 \\ 3 & 0 & -9 \end{vmatrix} \right\| \\ &= \frac{1}{2} \left| \mathbf{i} \begin{vmatrix} 0 & -2 \\ 0 & -9 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -1 & -2 \\ 3 & -9 \end{vmatrix} \right. \\ &\quad \left. + \mathbf{k} \begin{vmatrix} -1 & 0 \\ 3 & 0 \end{vmatrix} \right| \\ &= \frac{1}{2} \left\| \begin{vmatrix} -1 & -2 \\ +3 & -9 \end{vmatrix} \right\| \\ &= \left| \frac{1}{2}(9 + 6) \right| \\ &= \underline{\underline{\frac{15}{2}}} \end{aligned}$$