Question

Answer

(i) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 0$

Try auxiliary equation:

$m^2 - 3m + 2 = 0$

$\Rightarrow m = \dfrac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 2}}{2} = \dfrac{3 \pm 1}{2} = 2 \text{or } 1$

Therefore $y = Ae^{2x} + Be^{x}$ is a general solution. $A,\ B$ are constants to be found from boundary conditions.

(ii) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 2x^2 - 2x + 2$

$y = y_{comp.func.} + y_{partic.integral}$

(y_{CF} satisfies (i) so $y_{CF} = Ae^{2x} + Be^{x}$

y_{PI} is of the form $ax^2 + bx + c$: same degree of polynomial as RHS (as per lecture notes).

a, b, c to be found by substitution.

$$\begin{aligned} y_{PI} &= ax^2 + bx + c \\ y'_{PI} &= 2ax + b \\ y''_{PI} &= 2a \end{aligned}$$

In (ii)

$2a - 3(2ax + b) + 2(ax^2 + bx + c) = 2x^2 - 2x + 2$

$\Rightarrow 2ax^2 + (2b - 6a)x + 2a - 3b + 2c = 2x^2 - 2x + 2$

$$\begin{array}{lll} \text{Compare x}^2: & 2a = 2 & \Rightarrow\ a = 1 \\ \text{Compare x}^1: & 2b - 6a = -2 & \Rightarrow\ 2b - 6 = -2 \ \Rightarrow b = 2 \\ \text{Compare x}^0: & 2a - 3b + 2c = 2 & \Rightarrow\ 2 - 6 + 2c = 2 \Rightarrow c = 3 \end{array}$$

Therefore $y_{PI} = x^2 + 2x + 3$

Therefore $y_{general} = Ae^{2x} + Be^{x} + x^2 + 2x + 3$

Use boundary conditions to find A and B.

$y = 2$ when $x = 0$:

$$2 = Ae^{2\times 0} + Be^{0} + 3 = A + B + 3$$

$y' = 1$ when $x = 0$:

$$1 = 2Ae^{2\times 0} + Be^{0} + 2 \times 0 + 2 = 2A + B + 2$$

Therefore

$$\begin{array}{l} \ A + B = -1 \quad - \\ \underline{2A + B = -1} \\ -A \qquad = \quad 0 \end{array}$$

$\Rightarrow A = 0,\ B = -1$

Therefore $y = x^2 + 2x + 3 - e^x$.