Question

State the order and degree of the following differential equations, identify their type and hence solve them.

(i) $\dfrac{dy}{dx} = x^2 y$, where $y = 1$ when $x = 1$;

(ii) $\dfrac{dy}{dx} - 3x^2 y = e^{x^3} \cos x$, where $y = 1$ when $x = 0$.

Answer

(i) $\dfrac{dy}{dx} = x^2 y$: 1st order, 1st degree, variables separable

$\Rightarrow \displaystyle\int \frac{dy}{dx} = \int x^2\, dx$

$\Rightarrow \ln y = \dfrac{x^3}{3} + c$

But $y = 1$ when $x = 1$

$\Rightarrow$

$$\begin{aligned} \ln 1 &= \frac{1}{3} + c \\ 0 &= \frac{1}{3} + c \\ c &= -\frac{1}{3} \end{aligned}$$

$\Rightarrow \ln y = \dfrac{x^3}{3} - \dfrac{1}{3}$

or $y = e^{\frac{1}{3}(x^3 - 1)}$

$e^{-\frac{1}{3}} = 0.7165$

(ii) $\dfrac{dy}{dx} - 3x^2y = e^{x^3}\cos x$: 1st order, 1st degree.

Linear, requiring integrating factor (or exact)

Integrating factor $= e^{\int -3x^2\,dx} = e^{-x^3}$

Multiply through by it: $e^{-x^3}\dfrac{dy}{dx} - 3x^2e^{-x^2}y = e^{-x^3}e^{x^3}\cos x$

$\Rightarrow \dfrac{d}{dx}\left\{ye^{-x^3}\right\} = \cos x$

$\Rightarrow e^{-x^3}y = \displaystyle\int \cos x\,dx + c$

$\Rightarrow y = e^{x^3}[\sin x + c]$

$y = 1$ when $x = 0$

so $1 = e^0[\sin 0 + c]$

$\Rightarrow c = 1$

Therefore $y = e^{x^3}(\sin x + 1)$