

Question

Show that $(f_a^2)''(p) = 0$ for a period-doubling point of f_a .

Answer

$f_a^2(p) = f_a(f_a(p))$ so $(f_a^2)'(p) = f_a'(f_a(p)) \cdot f_a'(p)$.

Then $(f_a^2)''(p) = f_a''(f_a(p)) \cdot (f_a'(p))^2 + f_a'(f_a(p)) \cdot f_a''(p)$.

But $f_a(p) = p$ and $f_a'(p) = -1$, giving $(f_a^2)''(p) = 0$.