Question

Find the fixed points and a period 2 orbit for $f(x) = 2x^2 - 5x$. Decide in each case if they are attracting or repelling.

Answer

$f(x) = 2x^2 - 5x$. Fixed points where $2x^2 - 5x = x : \underline{x} = 0, 3$. $f^2(x) = 2(2x^2 - 5x)^2 - 5(2x^2 - 5x) = (2x^2 - 5x)(2(2x^2 - 5x) - 5) = x(2x - 5)(4x^2 - 10x - 5)$. Fixed points of f^2 where $x(2x-5)(4x^2-10x-5) = x$, i.e. $x^3 - 5x^2 + 5x + 3 = 0$. We know $x = 3$ is a solution, so $(x - 3)$ is a factor of LHS:

$$x^3 - 5x^2 + 5x + 3 = (x - 3)(x^2 - 2x - 1) = 0$$

giving $x = 1 \pm \sqrt{2}$.
(Check $f(1 \pm \sqrt{2}) = 2(1 \pm \sqrt{2})^2 - 5(1 \pm \sqrt{2}) = 2(3 \pm 2\sqrt{2}) - 5(1 \pm \sqrt{2}) = (1 \pm \sqrt{2})$.)
ALSO: $f'(x) = 4x - 5$ so fixed points repelling; $f'(1 + \sqrt{2})f'(1 - \sqrt{2}) = -31$ so 2-cycle repelling too.