Question

Show that $x \mapsto \cos x$ has a unique attracting fixed point, and no other periodic points. What about $x \mapsto \sin x$?

Answer

Graph of $y = \cos x$ meets the diagonal $y = x$ at one point; we have $\left|\frac{d}{dx}\cos x\right| = |\sin x| < 1$ when $x \neq \frac{n\pi}{2}$ so the fixed point is attracting. Graph of $y = \sin x$ meets the diagonal only at $x = 0$, with $|\sin(x)| < |x|$ for nonzero x: thus 0 is attracting.(Compare to $x - x^3$)