Question

Write down the proof that if the fixed point p for $f: \mathbf{R} \longrightarrow \mathbf{R}$ has $|f'(p)| > 1$ then p is a source.

Answer

Choose c with $|f'(p)| > c > 1$. Since by definition $f'(p) = \lim_{x \to p} \dfrac{f(x) - f(p)}{x - p}$ there is some $N_\epsilon(p)$ with $\left| \dfrac{f(x) - p}{x - p} \right| > c$ for all $x \leq N_\epsilon(p)$, $x \neq p$, i.e. $|f(x) - p| > c|x - p|$ for all $x \in N_\epsilon(p)$. Thus if $x, f(x), ..., f^{n-1}(x)$ all $\in N_\epsilon(p)$ we have $|f^n(x) - p| > c^n|x - p|$ so eventually $f^n(x) \notin N_\epsilon(p)$.

Therefore p is a source.