QUESTION

Find a solution of $x^2 + 1 \equiv 0$ mod 17.

ANSWER

The alert will spot ± 4 as roots immediately! if you didn't notice, then as $17 \equiv 1$ mod 4, we can appeal to the method of th.4.6 to deduce that the roots are $\pm\left(\frac{(p-1)}{2}\right)!$ where $p = 17$. Thus the roots are $\pm(8!)$ mod 17.

$8! = 8.7.6.5.4.3.2 = 56.30.24 \equiv 5.(-4).7 \equiv 5.(-28) \equiv 5.6 \equiv 30 \equiv -4$ mod 17, showing that the roots are ± 4 mod 17, as spotted!