QUESTION

Prove $n \geq 5$ is a prime if and only if $6(n-4)! \equiv 1 \bmod n$.

ANSWER

By Wilson's theorem (th.4.5), n is prime if and only if $(n-1)! \equiv -1 \bmod n$. Now $n-1 \equiv -1 \bmod n$, $n-2 \equiv -2 \bmod n$ and $n-3 \equiv -3 \bmod n$. Hence $(n-1)! \equiv (n-4)!(-3)(-2)(-1) \equiv -6(n-4)! \bmod n$. Thus n is prime if and only if $-6(n-4)! \equiv -1 \bmod n$, which holds if and only if $6(n-4)! \equiv 1 \bmod n$.