

QUESTION

Find the least positive residue of $3^{67} \pmod{31}$.

ANSWER

As 31 is prime, and $\gcd(3,31)=1$, Fermat's little theorem (Th.4.2) gives $3^{30} \equiv 1 \pmod{31}$.

Thus $3^{67} = 3^3 \cdot 3^{30} \cdot 3^7 \equiv 1 \cdot 1 \cdot 3^7 \pmod{31}$. But $3^3 = 27 \equiv -4 \pmod{31}$, and so $3^7 \equiv (-4) \cdot (-4) \cdot 3 \equiv 48 \equiv 17 \pmod{31}$. Thus $3^{67} \equiv 17 \pmod{31}$.