Question

At Las Vegas, a man with $20 needs $40, and hopes to raise the money by playing roulette. He is considering two strategies: bet $20 on 'evens' all at once and stop if he wins or loses, or bet on 'evens' one dollar at a time until he has won or lost $20. Which would you advise?

(Assume the roulette wheel has numbers $0, 0, 1, 2, \ldots, 36$ all equally likely to occur).

Answer

If he bets $20 all in one go the probability of winning is $\dfrac{18}{38} = 0.47\ldots$

If he bets $1 at a time until either he loses or wins $40, this can be regarded as a gambler's ruin problem, with

$$z = 20 \quad a = 40 \quad p = \frac{18}{38} \quad q = \frac{20}{38}$$

So the probability that he wins is the same as that of his adversary being ruined i.e.

$$p_z = \frac{1 - \left(\frac{q}{p}\right)^z}{1 - \left(\frac{q}{p}\right)^a} = \frac{1 - \left(\frac{20}{18}\right)^{20}}{1 - \left(\frac{20}{18}\right)^{40}} = \frac{1}{1 + \left(\frac{20}{18}\right)^{20}} = 0.108\ldots$$

So he is much less likely to win $40 by betting $1 at a time than by betting all $20 in one go.