Question

Sudden dips in a supply of electrical current in occur in accordance with the assumptions of a Poisson process with a rate of 3 dips per hour. What is the probability that the first dip in the afternoon occurs later than 1.00 p.m.? If a device fails when it has experienced the cumulative effect of 27 dips in the current, what is the probability density function of the life of the device? Find approximately the probability that such a device will last for longer than 12 hours.

Answer

Dips $\sim$ Poisson(3)

P(1st afternoon dip occurs after 1p.m.) $= P(W_1 > 1\text{hour}) = e^{-3} = 0.0498$ ($=P$(no events))

The life of a device = waiting time until 27th dip, and so has p.d.f.

$$\frac{3e^{-3t}(3t)^{26}}{26!} \qquad t \geq 0 \qquad (\Gamma(3, 27))$$

Now $\mu = \dfrac{27}{3} = 9$ and $\sigma^2 = \dfrac{27}{9} = 3$

So the lifetime $\sim N(9, 3)$

$P(L > 12) = P\left(Z > \dfrac{12-9}{\sqrt{3}}\right) = P(Z > \sqrt{3}) \approx P(Z > 1.73) \approx 0.4182$ from tables.