

Question

A transistor in a machine fails has to be replaced on average twice a week according to a Poisson process. Replacement transistors are bought annually and kept in store for use during the year. How many transistors should be bought to ensure only a 5% chance of running out of replacement during the year?

Answer

Let $N(t)$ be the number of breakdowns in t weeks. This has a Poisson distribution with parameter $2t$.

We want to buy n components, where

$$P(N(52) > n) \leq 0.05$$

$$N(52) \sim P(104) \approx N(104, 104) (= N_p)$$

$$P(N_p > n) = P\left(Z > \frac{n - 104}{\sqrt{104}}\right) \leq 0.05$$

$$\text{so } \frac{n - 104}{\sqrt{104}} \geq 1.645 \quad n \geq 121$$