Question
In a branching chain the number of offspring of any individual has a binomial distribution with $n = 3,\ p = \frac{1}{2}$. Find the probability P of extinction.

Answer
$Z_i \sim B(3, \frac{1}{2})$
So the p.g.f. is $A(s) = \dfrac{1}{8} + \dfrac{3}{8}s + \dfrac{3}{8}s^2 + \dfrac{1}{8}s^3$
So we have to solve $s^3 + 3s^2 - 5s + 1 = 0$
i.e. $(s-1)(s^2 + 4s - 1) = 0$
$s = 1,\ -2 \pm \sqrt{5}$
So $P = \sqrt{5} - 2 = 0.236\ldots$