Question
A psychologist makes the following assumptions concerning the behaviour of mice subjected to a particular feeding schedule. For any particular trial 80% of the mice that went right in the previous experiment will go right in this trial, and 60% of those mice that went left in the previous experiment will go right in this trial. If 50% went right in the first trial, what would he predict for

(a) the second trial?

(b) the third trial?

(c) the thousandth trial?

Answer
The two states are right(R) and left(L)

$$\text{Transition matrix } P = \begin{array}{c c} & \begin{array}{cc} R & L \end{array} \\ \begin{array}{c} R \\ L \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{pmatrix} \end{array}$$

Initial Distribution $\mathbf{p}_0 = (0.5, 0.5)$

(i) $\mathbf{p}_1 = \mathbf{p}_0 P = (0.5, 0.5)\begin{pmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{pmatrix} = (0.7, 0.3)$

We predict that 70 % go right and 30% go left.

(ii) $\mathbf{p}_2 = \mathbf{p}_1 P = (0.7, 0.3)P = (0.74, 0.26)$

(iii) Now provided that $|p + q - 1| < 1$,

$$\begin{aligned} P^n &\to \frac{1}{2-p-q}\begin{pmatrix} 1-q & 1-p \\ 1-q & 1-p \end{pmatrix} \\ &= \frac{1}{2-.8-.4}\begin{pmatrix} 0.6 & 0.2 \\ 0.6 & 0.2 \end{pmatrix} \end{aligned}$$

So $\mathbf{p}_0 P^n \to \frac{1}{0.8}(0.5, 0.5)\begin{pmatrix} 0.6 & 0.2 \\ 0.6 & 0.2 \end{pmatrix} = (0.75, 0.25)$

So 75% go right for n large.