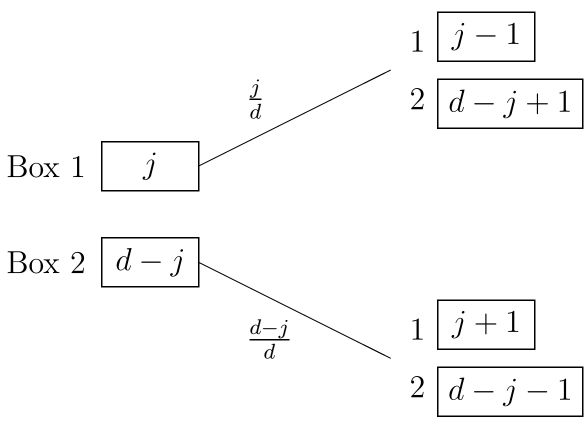
**Question**

The following is a simple model of the exchange the heat of or gas molecules between two isolated bodies known as the Ehrenfest model.

Suppose there are two boxes, labelled 1 and 2, and $d$ balls labelled $1, 2, \ldots, d$. Initially some of these balls are in the box 1 and the remainder are in box 2. An integer is selected at random from $1, 2, \ldots, d$, and the ball labelled by that integer is removed from its box and placed in the opposite box. This procedure is repeated indefinitely with the selections being independent from trail to trail. Let $X_n$ denote the number of balls in box 1 after the $n$-th trial. Check that $\{X_n\}$ is a Markov chain and find the 1-step transition probabilities.

**Answer**

Ehrenfest Diffusion Model




where $\frac{j}{d}$ is the prob. that the ball is taken from box 1

where $\frac{d-j}{d}$ is the prob. that the ball is taken from box 2

Let $X_n =$ number of balls in box 1 after the $n$-th trial.

This depends only on how many balls are in the box before the trial, and so is a Markov chain.

$$P(X_{n+1} = k | X_n = j) = \begin{cases} \frac{j}{d} & \text{if } k = j-1 \\ \frac{d-j}{d} & \text{if } k = j+1 \\ 0 & \text{otherwise} \end{cases}$$