

Question

From where he stands, one step toward the cliff would send the drunken man over the edge. He takes random steps either towards the cliff or away from the cliff. At any step his probability of taking a step away is $\frac{2}{3}$, or a step toward the cliff is $\frac{1}{3}$. What is his chance of escaping the cliff?

Answer

This problem is a simple random walk. with one absorbing barrier. It is equivalent to a gambler's ruin problem, played against an infinitely rich opponent with

$$z = 1 \quad p = \frac{2}{3} \quad \text{and} \quad q = \frac{1}{3}$$

The probability of ruin is $\left(\frac{q}{p}\right)^z = \frac{1}{2}$ in this case.