Question
Find the p.g.f. for a binomial $B(n,p)$ random variable, and use it to find the mean and variance.

Answer
For $j = 0$ to n, $p_j = \left(\begin{array}{c} n \\ j \end{array} \right) p^j q^{n-j} \quad (p+q) = 1$

So $G(s) = \sum_{j=0}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) p^j q^{n-j} s^j = (ps+q)^n$

$$\begin{array}{rclrcl} G'(s) & = & np(ps+q)^{n-1} & G'(1) & = & np = E(X) \\ G''(s) & = & n(n-1)p^2(ps+q)^{n-2} & G''(1) & = & n(n-1)p^2 \end{array}$$

$$\begin{aligned} VarX &= nn-1p^2 + np - (np)^2 \\ &= np - np^2 = np(p-1) = npq \end{aligned}$$