

**Question**

Let  $X$  be a Poisson random variable with parameter  $\lambda$ . Show that the p.g.f. for  $X$  is  $G(s) = e^{\lambda s - \lambda}$ .

Use this to find the mean and variance of  $X$ .

**Answer**

For a Poisson distribution with parameters  $\lambda$ ,

$$p_n = \frac{e^{-\lambda} \lambda^n}{n!}$$

So

$$G(s) = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n s^n}{n!} = e^{(\lambda s - \lambda)}$$

$$\begin{aligned} G'(s) &= \lambda e^{(\lambda s - \lambda)} & G'(1) &= \lambda = E(X) \\ G''(s) &= \lambda^2 e^{(\lambda s - \lambda)} & G''(1) &= \lambda^2 \end{aligned}$$

$$\text{Var}(X) = \lambda^2 + \lambda - \lambda^2 = \lambda$$