**Question**

Suppose that $S_t$, $t\epsilon(a,b)$ is a family of sets indexed by $t$. Suppose also that $t_1 < t_2 \Rightarrow S_{t_1} \subseteq S_{t_2}$. Prove that if $S_t$ is measurable for each $t\epsilon(a,b)$, then $\bigcup_{t\epsilon(a,b)} S_t$ is measurable, and that $m\left(\bigcup_{t\epsilon(a,b)} S_t\right) = \lim_{t\to b-} m(S_t)$

Formulate and prove a corresponding result involving intersections.

**Answer**

Let $\{t_n\}$ be an arbitrary increasing sequence converging to $b-$.

Then $S_{t_1} \subseteq S_{t_2} \subseteq \cdots \subseteq S_{t_n} \subseteq \cdots$

and so $\lim_{n\to\infty} m(S_{t_n}) = m\left(\bigcup_{n=1}^{\infty} S_{t_n}\right)$

Now $\lim_{n\to\infty} m(S_{t_n}) = \lim_{t\to b-} m(S_t)$

Since $t_n$ is an arbitrary sequence.

Also $\bigcup_{n=1}^{\infty} S_{t_n} = \bigcup_{t\epsilon(a,b)} S_t$, for $\subseteq$ obvious

if $x\epsilon \bigcup_{t\epsilon(a,b)} S_t$, there exists $t\epsilon(a,b)$, $x\epsilon S_t$

there exists $n$, with $t_n > t$, therefore $x\epsilon S_{t_n}$ therefore $x\epsilon \bigcup_{n=1}^{\infty} S_{t_n}$ therefore $\supseteq$.

Hence equality

Thus $m\left(\bigcup_{t\epsilon(a,b)} S_t\right) = \lim_{t\to b-} m(S_t)$

For intersections

$m\left(\bigcap_{t\epsilon(a,b)} S_t\right) = \lim_{t\to a+} m(S_t)$