QUESTION

(a) Using partial fractions evaluate $\int_1^2 \frac{1}{x(x+2)}\,dx$.

(b) **(i)** Show that

$$\frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x}.$$

(ii) Given that $x = 2$ is an approximate solution of the equation $2\tanh x = x$ use the Newton Raphson formula THREE times, and the result in part (i), to obtain a better approximation (correct to four decimal places).

ANSWER

(a) $\int_1^2 \frac{1}{x(x+2)}\,dx$

$\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)}$ therefore $1 = A(x+2) + Bx$.

$x = 0;\ 1 = 2A + 0,\ A = \frac{1}{2}$

$x = -2;\ 1 = 0 - 2B,\ B = -\frac{1}{2}$

therefore

$$\begin{aligned}
\int_1^2 \frac{1}{x(x+2)}\,dx &= \int_1^2 \left\{\frac{\frac{1}{2}}{x} - \frac{\frac{1}{2}}{x+2}\right\} dx \\
&= \left[\frac{1}{2}\ln x - \frac{1}{2}\ln(x+2)\right]_1^2 \\
&= \left(\frac{1}{2}\ln 2 - \frac{1}{2}\ln 4\right) - \left(\frac{1}{2}\ln 1 - \frac{1}{2}\ln 3\right) \\
&= \frac{1}{2}\ln\left(\frac{2\times 3}{4}\right) \\
&= \frac{1}{2}\ln\left(\frac{3}{2}\right)
\end{aligned}$$

(b) **(i)** Now $u = \tanh x = \frac{\sinh x}{\cosh x}$, $\frac{du}{dx} = \frac{\cosh x\cosh x - \sinh x \sinh x}{(\cosh x)^2} = \frac{1}{\cosh^2 x}$

(ii) $f(x) = 2\tanh x - x,\ f'(x) = 2\text{sech}^2 x - 1$
given $x_0 = 2$,

$x_1 = 2 - \dfrac{f(2)}{f'(2)} = 2 - \dfrac{-0.071945}{-0.85870} = 1.91622$
$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 1.91622 - \dfrac{-0.00101}{-0.83401} = 1.91501$
$x_3 = 1.91501 - \dfrac{f(1.191501)}{f'(1.91501)} = 1.91501 - \dfrac{-1.63 \times 10^{-6})}{(-0.83363)} = 1.91501$
Third approximation $\Rightarrow x = 1.9150$ (correct to 4 decimal places.)