Question

With respect to a set of orthogonal axes, origin O the coordinates of A, B, C, D are $(3,2,1)$, $(1,-2,1)$, $(-1,1,2)$ and $(-2,2,0)$ respectively.

Find the angle between OA and OD, the direction cosines of a vector perpendicular to OA and OD and the perpendicular distance from C to AB.

Find the equation of the plane through D parallel to the plane containing A, B and C.

Answer

$\mathbf{a} = (3,2,1) \quad \mathbf{b} = (1,-2,1) \quad \mathbf{c} - (-1,1,2) \quad \mathbf{d} = (-2,2,0)$

$|a| = \sqrt{14} \quad |d| = \sqrt{8} \quad \mathbf{a}\cdot\mathbf{d} = -2 \quad \cos\theta = -\dfrac{1}{\sqrt{28}} \Rightarrow \theta = 100^\circ,\ 1.761 rad$

$\mathbf{a}\times\mathbf{d} = (-2,-2,10) \quad |\mathbf{a}\times\mathbf{d}| = \sqrt{108} = 6\sqrt{3}$

direction cosines are $\left(-\dfrac{1}{3\sqrt{3}}, -\dfrac{1}{3\sqrt{3}}, \dfrac{5}{3\sqrt{3}}\right)$

Distances of c from the line AB is $\dfrac{|(c-a)\times|(b-a)|}{|b-a|} = \dfrac{|(4,-2,14)|}{2\sqrt{5}} = \dfrac{3\sqrt{6}}{\sqrt{5}}$

A normal to the plane is $(a-c)\times(a-b) = (4,-2,14)$ and the equation of the plane is $2x - y + 7z = -6$