NOTE REFERENCE TO QUESTION 10

Question

Verify the following integral.

$$\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} dz \frac{\exp(zt)}{\sqrt{z+1}} = \frac{\exp(-t)}{\sqrt{\pi t}},\ a>0$$

Answer

$$I = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{e^{zt}}{\sqrt{z+1}} dz \text{ put } z+1=\zeta \text{ say}$$

$$= \frac{1}{2\pi i}\int_{a-1-i\infty}^{a+1+i\infty} \frac{e^{\zeta t}}{\sqrt{\zeta}} e^{-t} d\zeta = e^{-t} \times \text{ integral of Q10}$$

Use same arguments as in Q10 and you get $I = \dfrac{e^{-t}}{\sqrt{\pi t}}$