Applications of Partial Differentiation
Extremes within restricted domains

Question

A building developer has bought a 10 hectare plot of land. On it he could build 6 detached houses per hectare, 8 pairs of semi-detached houses per hectare, or 12 flats per hectare.
The profits for each building are £40,000 per detached house, £20,000 per pair of semi-detached houses and £16,000 per flat.
However, council bylaws require that he build at least as many flats as detached houses or pairs of semi-detached house.
How many of each building should he build to maximize his profit?

Answer

If the developer builds x detached houses, y pairs of semi-detached houses and z flats then his profit will be

$$P = 40000x + 20000y + 16000z.$$

The imposed constraints mean that

$$\frac{x}{6} + \frac{y}{8} + \frac{z}{12} \leq 10 \Leftrightarrow 4x + 3y + 2z \leq 240$$

and

$$z \geq x + y.$$

Obviously $x \geq 0$, $y \geq 0$ and $z \geq 0$.
Now, the planes $4x + 3y + 2z = 240$ and $z = x + y$ intersect where $6x + 5y = 240$. This means that the constraint region has the vertices $(0, 0, 0)$, $(40, 0, 40)$, $(0, 48, 48)$ and $(0, 0, 120)$. These yield revenues of £0, £2240000, £1728000 and £1920000 respectively. So to maximize his profit, the developer should build 40 detached house and 40 flats, but no pairs of semi-detached houses.