Applications of Partial Differentiation
Extremes within restricted domains

Question

A cocktail bar sells two main drinks. The 'Vodka + Orange Fizz' has 20% vodka, 50% orange juice and 30% lemonade, and sells for £3 per pint. The 'Fizzy Orange + Vodka' has 10% vodka, 40% orange juice and 50% lemonade, and sells for £2 per pint.

If the bar has 2000 pints of vodka and 6000 pints of both orange juice and of lemonade, how many pints of each cocktail should be made to maximize the revenue?

Answer

Suppose that x pints of the first cocktail and y pints of the second cocktail are made, then the total revenue will be

$$R = 3x + 2y.$$

The constraints imposed by the availability of ingredients are

$$\begin{array}{rlcrl} \frac{20}{100}x + \frac{10}{100}y & \le 2000 & \Leftrightarrow & 2x + y & \le 20000 \\ \frac{50}{100}x + \frac{40}{100}y & \le 6000 & \Leftrightarrow & 5x + 4y & \le 60000 \\ \frac{30}{100}x + \frac{50}{100}y & \le 6000 & \Leftrightarrow & 3x + 5y & \le 60000 \end{array}$$

The lines $2x + y = 20000$ and $5x + 4y = 60000$ intersect at the point $\left(\frac{20000}{3}, \frac{20000}{3}\right)$. This satisfies $3x + 5y \le 60000$, so it lies in the constraint region, and

$$f\left(\frac{20000}{3}, \frac{20000}{3}\right) \approx 33333.$$

The lines $2x + y = 20000$ and $3x + 5y = 60000$ intersect at the point $\left(\frac{40000}{7}, \frac{60000}{7}\right)$. This doesn't satisfy $5x + 4y \le 60000$, so doesn't lie in the constraint region.

The lines $5x + 4y = 60000$ and $3x + 5y = 60000$ intersect at the point $\left(\frac{60000}{13}, \frac{120000}{13}\right)$. This satisfies $2x + y \le 20000$, so it lies in the constraint region, and

$$f\left(\frac{60000}{13}, \frac{120000}{13}\right) \approx 32307.$$

This means that to produce the maximum possible revenue, the bar should make $20000/3 \approx 6667$pints of each cocktail.