Applications of Partial Differentiation
Extremes within restricted domains

Question
$F(x, y, z) = 2x + 3y + 4z$
Minimize F, subject to

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ z &\geq 0 \\ x + y &\geq 2 \\ y + z &\geq 2 \\ x + z &\geq 2 \end{aligned}$$

Answer
The constraint region has vertices $(1, 1, 1)$, $(2, 2, 0)$, $(2, 0, 2)$ and $(0, 2, 2)$. This gives

$$F(1, 1, 1) = 9, \quad F(2, 2, 0) = 10, \quad F(2, 0, 2) = 12, \quad F(0, 2, 2) = 14$$

And so the minimum value of F under the given constraints is $F = 9$.