Applications of Partial Differentiation
Extremes within restricted domains

Question
$f(x,y) = xye^{-xy}$ has domain $x > 0$, $y > 0$.
Show that $\lim_{x\to\infty} f(x,kx) = 0$.
Is there a limit for f as (x,y) moves arbitrarily far from the origin yet still remaining in the first quadrant?
Is there a maximum value for f in the given domain?
Answer

$$\begin{aligned} f(x,kx) &= kx^2e^{-kx^2} \to 0 \\ \text{as } x \to \infty & \quad \text{if } k > 0 \\ \text{and } f(x,0) &= f(0,y) = 0 \end{aligned}$$

$\Rightarrow f(x,y) \to 0$ as $(x,y) \to \infty$ along any straight line from the origin in Q.
But $f\left(x, \frac{1}{x}\right)$ and $f(x,0) = 0$, $\forall x > 0$, even though $\left(x, \frac{1}{x}\right)$ and $(x,0)$ move arbitrarily close as x increases. And so f has no limit as $x^2 + y^2 \to \infty$.
See that $f(x,y) = re^{-r} = g(r)$ on the hyperbola $xy = r > 0$.

$$\begin{aligned} g(r) &\to 0 \\ \text{as } r &\to 0, \quad \text{or } r \to \infty \\ \text{and } g'(r) &= (1-r)e^{-r} = 0 \\ \Rightarrow r &= 1 \end{aligned}$$

And so $f(x,y)$ is less than $g(1) = 1/e$ everywhere on Q.
$\Rightarrow f$ has a maximum value on Q.