

Applications of Partial Differentiation
Extremes within restricted domains

Question

Find the maximum and minimum values of

$$f(x, y) = xy(1 - x - y)$$

Over the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$.

Answer

It can easily be seen that $f(x, y) = 0$ on all three of the boundary segments, and that $f(x, y) > 0$ inside the triangle, therefore maximum value of f must occur at a critical point inside the triangle.

For critical points

$$0 = f_1(x, y) = y(1 - 2x - y)$$

$$0 = f_2(x, y) = x(1 - x - 2y)$$

And so the only critical points are $(0, 0)$, $(1, 0)$ and $(0, 1)$ and $(1/3, 1/3)$. These are all on the boundary of the triangle, except for $(1/3, 1/3)$ which is inside.

The maximum value of f over the triangle is

$$f(1/3, 1/3) = \frac{1}{27}$$