Applications of Partial Differentiation
Extremes within restricted domains

Question
Find the maximum and minimum values of

$$f(x, y) = x + 2y$$

On the disk $x^2 + y^2 \leq 1$.
Answer
As $f_1 = 1$ and $f_2 = 2$, f will have no critical points. So the minimum and maximum points must occur at the boundary. f must have minimum and maximum points as f is a continuous on a closed, bounded set in the plane. The boundary is the circle $x^2 + y^2 = 1$ and can be parameterised as

$$\begin{aligned} x &= \cos t \\ y &= \sin t \end{aligned}$$

$$\Rightarrow f(x, y) = f(\cos t, \sin t) = \cos t + 2 \sin t = g(t)$$

For critical points of g

$$\begin{aligned} 0 &= g'(t) = -\sin t + 2 \cos t \\ \Rightarrow \tan t &= 2 \\ x &= \pm 1/\sqrt{5} \\ y &= \pm 2/\sqrt{5} \end{aligned}$$

So the critical points of g are

$$\begin{aligned} (-1/\sqrt{5}, -2/\sqrt{5}) \qquad & f = -\sqrt{5} = \min(f) \\ (1/\sqrt{5}, 2/sqrt5) \qquad & f = \sqrt{5}\max(f) \end{aligned}$$