Applications of Partial Differentiation
Extremes within restricted domains

Question
Find the maximum and minimum values of

$$f(x, y) = xy - 2x$$

On the rectangle $-1 \leq x \leq 1$, $0 \leq y \leq 1$.
Answer
For critical points

$$\begin{aligned} 0 &= f_1(x, y) = y - 2 \\ 0 &= f_2(x, y) = x \end{aligned}$$

So the only critical point is $(0, 2)$, this lies outside of the rectangle.
This implies that the minimum and maximum values of f lie on the four boundary segments of the rectangle.
On $x = -1$

$$\begin{aligned} f(-1, y) &= 2 - y \\ \text{for } 0 \leq y \leq 1 \end{aligned}$$

This has min=1 and max=2.
On $x = 1$

$$\begin{aligned} f(1, y) &= y - 2 \\ \text{for } 0 \leq y \leq 1 \end{aligned}$$

This has min=-2 and max=-1.
On $y = 0$

$$\begin{aligned} f(x, 0) &= -2x \\ \text{for } -1 \leq x \leq 1 \end{aligned}$$

This has min=-2 and max=2.
On $y = 1$

$$\begin{aligned} f(x, 1) &= -x \\ \text{for } -1 \leq x \leq 1 \end{aligned}$$

This has min=-1 and max=1.
So for f on the rectangle,

$$\begin{aligned} \max(f) &= -2 \\ \max(f) &= 2 \end{aligned}$$