

Question

Arrange the following in descending order for small positive ε :

(a) $\varepsilon^2, \varepsilon^{\frac{1}{2}}, \log\left(\log\frac{1}{\varepsilon}\right), 1, \varepsilon^{\frac{1}{2}}\log\left(\frac{1}{\varepsilon}\right),$
 $\varepsilon\log\left(\frac{1}{\varepsilon}\right), e^{-\frac{1}{\varepsilon}}, \log\left(\frac{1}{\varepsilon}\right), \varepsilon^{\frac{3}{2}}, \varepsilon, \varepsilon^2\log\left(\frac{1}{\varepsilon}\right).$

(b) $e^{-\frac{1}{\varepsilon}}, \log\left(\frac{1}{\varepsilon}\right), \varepsilon^{-0.01}, \cot\varepsilon, \sinh\left(\frac{1}{\varepsilon}\right).$

Answer

Try them on your calculator for $\varepsilon \rightarrow 0+$ and then use limits to justify them

(a) $\log\left(\frac{1}{\varepsilon}\right) > \log\left(\log\frac{1}{\varepsilon}\right) > 1 > \varepsilon^{\frac{1}{2}}\log\left(\frac{1}{\varepsilon}\right) > \varepsilon^{\frac{1}{2}} > \varepsilon\log\left(\frac{1}{\varepsilon}\right) > \varepsilon > \varepsilon^{\frac{3}{2}} >$
 $\varepsilon^2\log\left(\frac{1}{\varepsilon}\right) > \varepsilon^2 > e^{-\frac{1}{\varepsilon}}$

(b) $\sinh\left(\frac{1}{\varepsilon}\right) > \cot\varepsilon > \varepsilon^{-0.01} > e^{-\frac{1}{\varepsilon}} > \log\left(\frac{1}{\varepsilon}\right) > e^{-\frac{1}{\varepsilon}}$