

**Question**

Show that if  $f(x) \sim g(x)$  as  $x \rightarrow \infty$ , then  $f(x) = \{1 + o(1)\}g(x)$  as  $x \rightarrow \infty$ .

**Answer**

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1 \Leftrightarrow f \sim g, \quad x \rightarrow \infty$$

$$\text{Thus } \lim_{x \rightarrow \infty} \left( \frac{f(x)}{g(x)} - 1 \right) = 0$$

$$\Rightarrow \frac{f(x)}{g(x)} - 1 = o(1) \Rightarrow \underline{f(x) = [1 + o(1)]g(x)}$$