

QUESTION

Find the residue at $z = 0$ of each of the functions (a) $\frac{1}{z^3}$, (b) $\frac{1}{z+z^2}$, (c) $\frac{z-\sin z}{z}$, (d) $\frac{\sinh z}{z^4(1-z^2)}$.

ANSWER

The residue at $z = 0$ is the coefficient of $\frac{1}{z}$ in the Laurent expansion about $z = 0$. (a) 0, (b) there is a simple pole at $z = 0$ so its residue is $\lim_{z \rightarrow 0} \frac{z}{z+z^2} = 1$, (c) This function has a removable singularity at $z = 0$ so that the residue at $z = 0$ is 0. (d) Using Taylor expansions about $z = 0$ we get $\frac{1}{z^4}(z + z^3/3! + \dots)(1 + z^2 + z^4 + \dots)$. The coefficient of $\frac{1}{z}$ is $7/6$.