

QUESTION

Show that $f(z)$ has a pole of order m at z_0 if and only if $\frac{1}{f(z)}$ has a zero of order m at z_0 . Thus find the order of the pole of $1/z(e^z - 1)$ at $z = 0$.

ANSWER

$f(z)$ has a pole of order m at z_0 if and only if near z_0 we have $f(z) = (z - z_0)^{-m}g(z)$ where g is analytic at z_0 and $g(z_0) \neq 0$. Thus $\frac{1}{f(z)} = (z - z_0)^m/g(z)$ has a zero of order m at $z = z_0$.