QUESTION

(From 1996 exam.) Classify the singularities of the function

$$\frac{cos(z\pi/2)sin(z^{-1})}{(z^2-1)(z-2).}$$

ANSWER

The points z where singularities may occur are $z = 0, 1, -1, 2$. When $z = 0$ there must be an essential singularity as $sin(z^{-1})$ has a Laurent expansion about $z = 0$ with an infinite number of negative powers of z. At $z = 2$ there is a simple pole. As $\cos(\pi/2) = \cos(-\pi/2) = 0$ w need to take care of the points $z = 1$ and $z = -1$. If, for example, we use L'Hôpital's rule, we find that the limit as $z \to 1$ of $\frac{\cos(z\pi/2)}{z^2-1}$ is finite and also for the limit as $z \to -1$. Thus there are removable singularities when $z = 1$ and $z = -1$.