Exam Question
Topic: Tangent Plane
Find an equation for the plane which is tangent to the surface whose equation is $z = x^2y^3 - 3(x + y^2)$ at the point $P(1, -1, 7)$.
Express the equation in the form $ax+by+cz = d$, where a, b, c, d are constants
the line L is a horizontal tangent line (parallel to the x-y plane) to the surface at the point $P(1, -1, 7)$.
Find the angle which L makes with the x-direction
Express your answer in radians correct to 3 decimal places using your calculator.

Solution
$z = f(x, y) = x^2y^3 - 3(x + y^2)$
$f_x = 2xy^3 - 3; \quad f_x(1, -1) = -5.$
$f_y = 3x^2y^2 - 6y; \quad f_y(1, -1) = 9.$
The equation of the tangent plane is therefore
$z + 7 = -5(x - 1) + 9(y + 1); \quad z + 7 = -5x + 5 + 9y + 9; \quad 5x - 9y + z = 7.$
the directional derivative at P in direction θ is given by

$$D(\theta) = -5\cos\theta + 9\sin\theta.$$

This is zero when $\tan\theta = 5/9; \quad \theta = 0.507$ radians (3 d.p.)