Exam Question
Topic: Chain Rule

Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ where

$$z = \sin(xy) + x\cos y.$$

A change of variables is specified by means of the equations

$$x = u^2 + v^2, \quad y = uv.$$

Use the chain rule to find $\dfrac{\partial z}{\partial u}$ in terms of u and v.
Calculate the value of this partial derivative when $u = 1,\ v = 0$.

Solution

$$\begin{aligned} z &= \sin(xy) + x\cos y \\ \frac{\partial z}{\partial x} &= y\cos(xy) + \cos y \\ \frac{\partial z}{\partial y} &= x\cos(xy) - x\sin y \end{aligned}$$

The chain rule gives

$$\begin{aligned} \frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \\ &= \Big(uv\cos\Big((u^2+v^2)uv\Big) + \cos(uv)\Big).2u \\ &+ \Big((u^2+v^2)\cos\Big((u^2+v^2)uv\Big) - (u^2+v^2)\sin(uv)\Big).v \end{aligned}$$

When $u = 1,\ v = 0$ the value of this expression is 2.