Question

Calculate the determinants of the following matrices.

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ 2 & 3 & 1 \end{pmatrix}; \; B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 3 & -1 & -1 \end{pmatrix}.$$

Calculate the determinants of the matrix AB and verify that it is equal to $\det(A)\det(B)$.

Answer

$$\det(A) = \begin{vmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = (1)\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 0 = (-1-3) + (1-2) = -5$$

$$\det(B) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 3 & -1 & -1 \end{vmatrix} = (1)\begin{vmatrix} -2 & 1 \\ -1 & -1 \end{vmatrix} - 0 + 0 = (2+1) = 3$$

$$AB = \begin{pmatrix} 1 & 2 & -1 \\ 4 & 1 & -2 \\ 5 & -7 & 2 \end{pmatrix}$$

so that

$$\begin{aligned}
\det(AB) = \begin{vmatrix} 1 & 2 & -1 \\ 4 & 1 & -2 \\ 5 & -7 & 2 \end{vmatrix} &= (1)\begin{vmatrix} 1 & -2 \\ -7 & 2 \end{vmatrix} - (2)\begin{vmatrix} 4 & -2 \\ 5 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 4 & 1 \\ 5 & -7 \end{vmatrix} \\
&= (2-14) - (2)(8+10) - (-28-5) \\
&= -15 \\
&= (-5)(3) \\
&= \det(A)\det(B).
\end{aligned}$$