Question

Decide which of the following matrices can be added, and which can be multiplied. Carry out the calculations whenever possible.

$$A = \begin{pmatrix} 1 & -1 & -2 \\ 0 & 2 & 1 \end{pmatrix};\ B = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 1 & -3 \end{pmatrix};\ C = \begin{pmatrix} 4 & -1 & 0 \\ 3 & -2 & 1 \\ 5 & -6 & -7 \end{pmatrix};\ D = \begin{pmatrix} 3 & -4 & 7 \\ -2 & 1 & 6 \end{pmatrix}.$$

For each of the matrices, write down its transpose and say which of the transposed matrices can be multiplied.

Answer

Any matrix can be added to itself: this has the effect of doubling each entry. For example:

$$A + A = \begin{pmatrix} 2 & -2 & -4 \\ 0 & 4 & 2 \end{pmatrix}$$

Otherwise, only A and D can be added, with

$$A + D = \begin{pmatrix} 4 & -5 & 5 \\ -2 & 3 & 7 \end{pmatrix}.$$

Possibilities for multiplication are:

$$AB = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix};\quad AC = \begin{pmatrix} -9 & 13 & 13 \\ 11 & -10 & -5 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & -3 & -3 \\ 0 & 4 & 2 \\ 1 & -7 & -5 \end{pmatrix};\quad BD = \begin{pmatrix} 5 & -5 & 1 \\ -4 & 2 & 12 \\ 9 & -7 & -11 \end{pmatrix}$$

$$CB = \begin{pmatrix} 4 & -6 \\ 4 & -10 \\ -2 & 4 \end{pmatrix};\quad CC = C^2 = \begin{pmatrix} 13 & -2 & -1 \\ 11 & -5 & -9 \\ -33 & 49 & 43 \end{pmatrix}$$

$$DB = \begin{pmatrix} 10 & -32 \\ 4 & -14 \end{pmatrix};\quad DC = \begin{pmatrix} 35 & -37 & -53 \\ 25 & -36 & -41 \end{pmatrix}$$

Transposed matrices:

$$A^T = \begin{pmatrix} 1 & 0 \\ -1 & 2 \\ -2 & 1 \end{pmatrix};\ B^T = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & -3 \end{pmatrix};$$

$$C^T = \begin{pmatrix} 4 & 3 & 5 \\ -1 & -2 & -6 \\ 0 & 1 & -7 \end{pmatrix}; \; D^T = \begin{pmatrix} 3 & -2 \\ -4 & 1 \\ 7 & 6 \end{pmatrix}$$

Possibilities for multiplying:

$$A^T B^T, \; B^T A^T, \; B^T C^T, \; B^T D^T, \; C^T A^T, \; C^T C^T, \; D^T B^T, \; C^T D^T.$$

<u>Note</u>: $A^T B^T = (BA)^T = \begin{pmatrix} 1 & -3 & -3 \\ 0 & 4 & 2 \\ 1 & -7 & -5 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 1 \\ -3 & 4 & -7 \\ -3 & 2 & -5 \end{pmatrix}$