Question

Let

$$T(z) = \frac{az+b}{cz+d} \qquad a,b,c,d, \in \mathbf{R}, \qquad ad-bc=1$$

be a real Mobius transformation. Show that T maps the upper half plane to itself. Prove that the hyperbolic arc-length defined by

$$ds = \frac{|dz|}{y} \qquad\qquad (z = x+iy)$$

is invariant under all real Mobius transformations.

Show that $6+4i$ and $7+3i$ are at the same Euclidean distance from the point 3, and hence determine the hyperbolic line which passes through $6+4i$ and $7+3i$. By finding a Mobius transformation which maps this hyperbolic line to the imaginary axis compute the hyperbolic distance from $6+4i$ to $7+3i$.

Answer

1st bit only in this years course

T maps the real axis to the real axis

$\text{im}T(i) = \text{im}\dfrac{ai+b}{ci+d} = \text{im}\dfrac{(ai+b)(-ci+d)}{c^2+d^2} = \dfrac{ad-bc}{c^2+d^2} > 0$