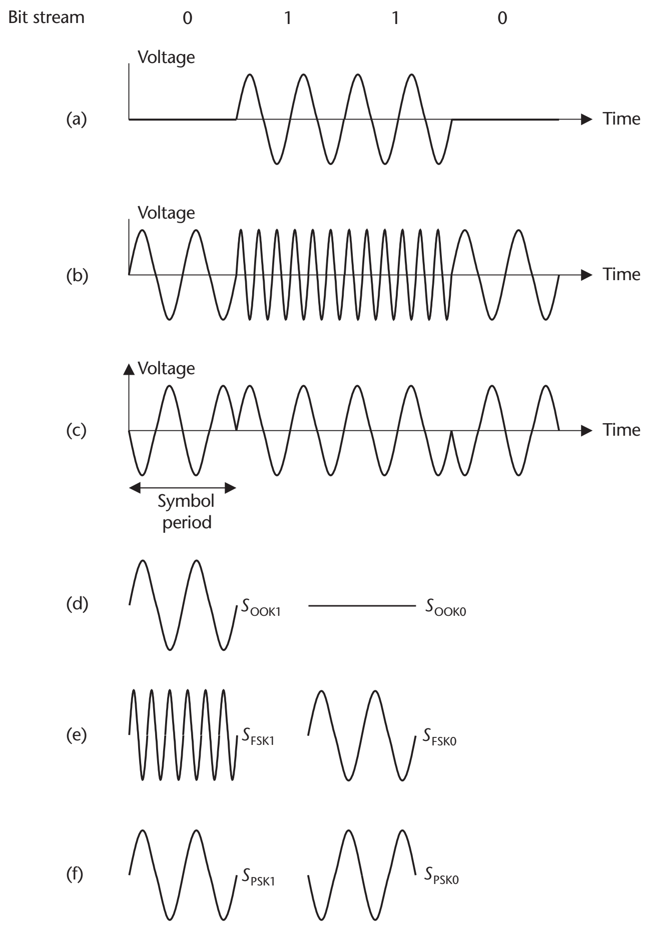
ELEC1323 Communications

6 Digital Carrier Modulation

Digital carrier modulation - Binary shift keying

Figure 1.30



- (a) and (d) On-Off Keying (OOK) is a special case of Binary Amplitude Shift Keying (BASK), in which different carrier amplitudes A are used for the duration of each symbol period depending on the corresponding bit value.

- (b) and (e) Binary Frequency Shift Keying (BFSK) uses different carrier frequencies f_c depending on the bit values.

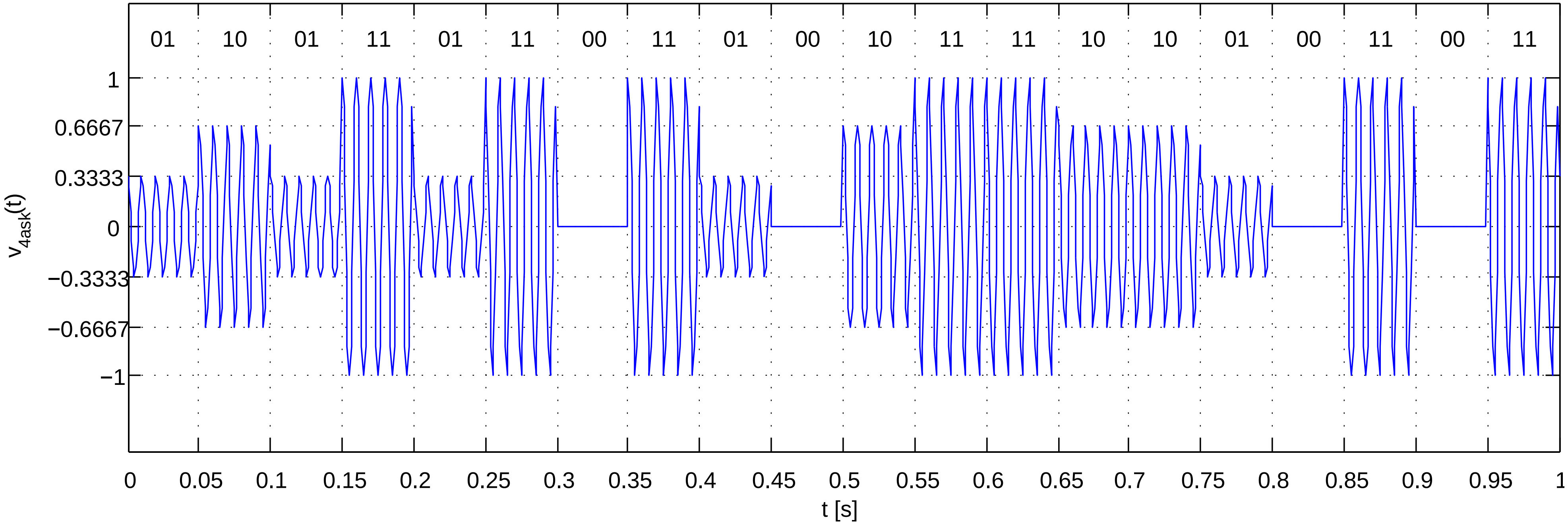
- (c) and (f) Binary Phase Shift Keying (BPSK) uses values of 0 and π radians for the phase of the carrier ϕ depending on the bit values.

Digital carrier modulation - M-ary shift keying (CEP 7.9)

- Binary shift keying uses $k = 1$ bit per symbol and $M = 2$ values for the modulated parameter of the carrier.
- In general, we can use any integer number k of bits per symbol. We just need a modulation scheme that uses $M = 2^k$ values for the modulated parameter of the carrier. i.e. M number of amplitudes, phases or frequencies.
- The symbol rate R_s is k times slower than the bit rate R_b, i.e. $R_s = R_b/k$.

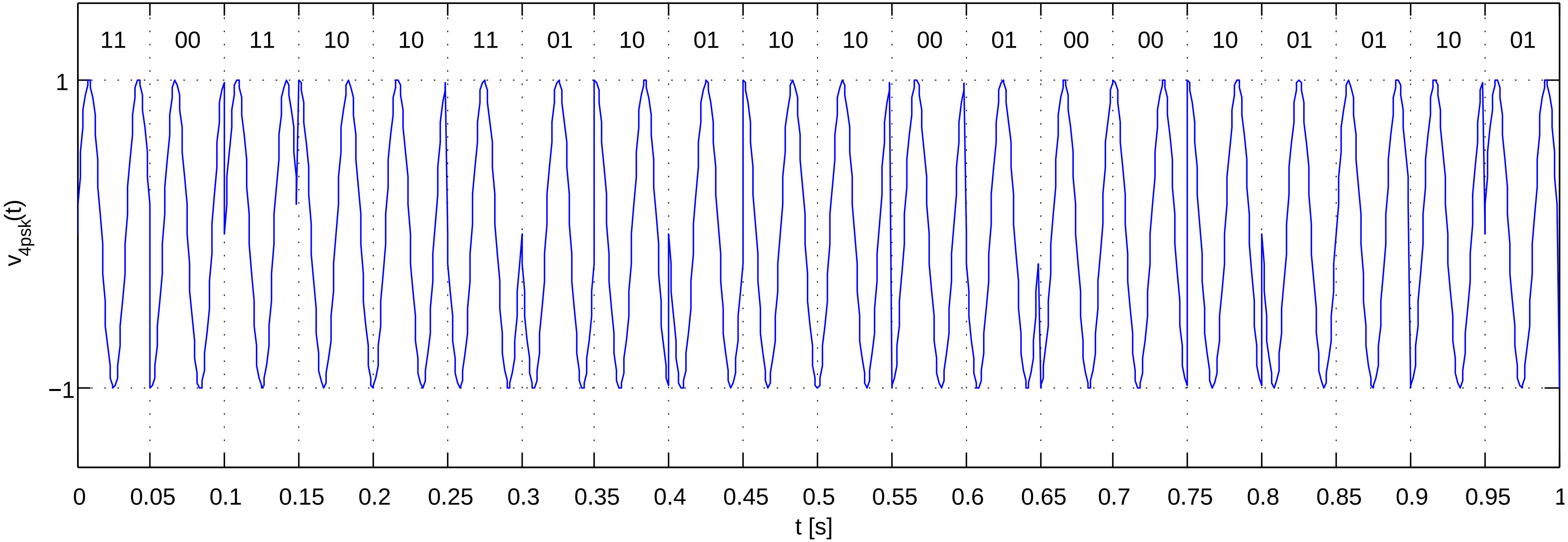
Digital carrier modulation - 4-ary ASK (CEP 7.9.2)

- M-ary ASK can use values of $\{m/(M-1)\}_{m=0}^{M-1}$ for the amplitude of the carrier A.
- Similar to in quantization, we need a mapping between sequences of k bits and the M values of the modulated parameter of the carrier. e.g. in $M = 4$-ary ASK we may have 00 $\rightarrow 0$ V, 01 $\rightarrow 1/3$ V, 10 $\rightarrow 2/3$ V and 11 $\rightarrow 1$ V.



Digital carrier modulation - 4-ary PSK (CEP 7.9.3)

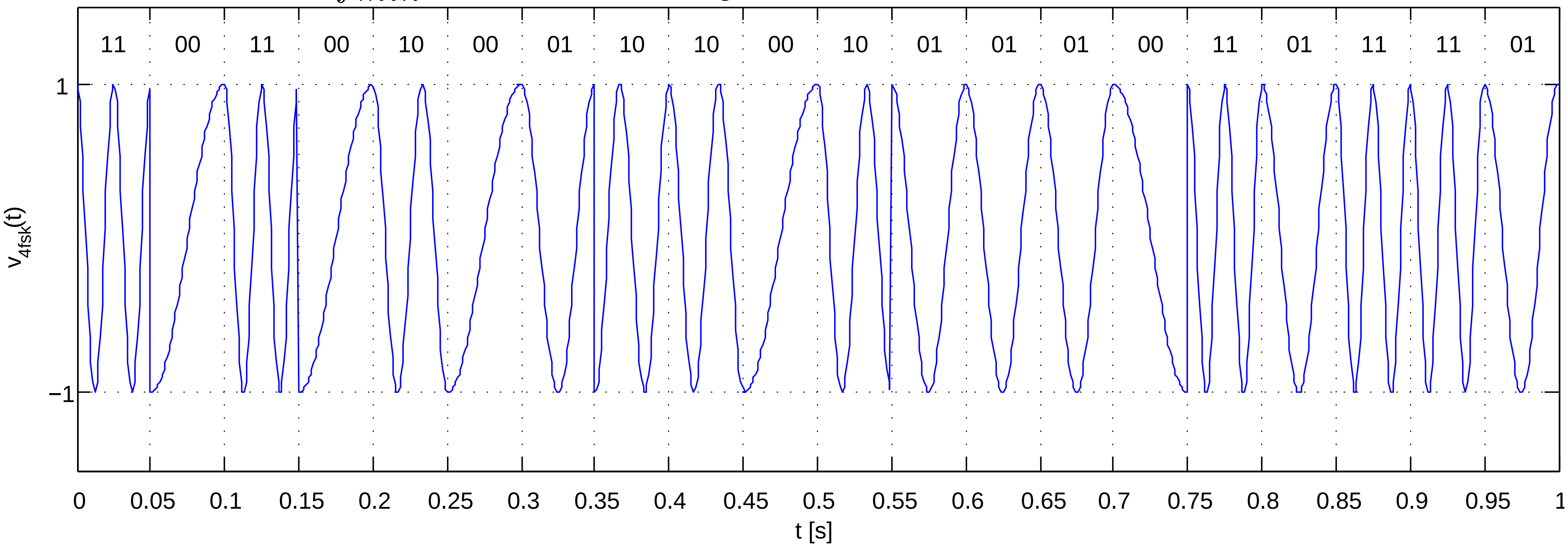
- M-ary PSK uses values of $\{2\pi m/M\}_{m=0}^{M-1}$ for the phase of the carrier ϕ.
- $M = 4$-ary PSK may use 00 $\rightarrow 0$ radians, 01 $\rightarrow \pi/2$ radians, 10 $\rightarrow \pi$ radians and 11 $\rightarrow 3\pi/2$ radians.



Digital carrier modulation - 4-ary FSK (CEP 7.9.4)

• M-ary FSK uses values of $\{f_{min} + 0.5R_s m\}_{m=0}^{M-1}$ for the frequency of the carrier f_c, where R_s is the symbol rate.

• $M = 4$-ary FSK may use 00 $\rightarrow 10$ Hz, 01 $\rightarrow 20$ Hz, 10 $\rightarrow 30$ Hz and 11 $\rightarrow 40$ Hz, in the case where $f_{min} = 10$ Hz and $R_s = 20$ Hz.



Multiplication of two sinusoids (CEP 2.3.4)

Trigonometric identity (1) from Lecture 4 shows what happens when you multiply two sinusoids having different frequencies

$$\begin{aligned} v(t) &= A_1 \cos(2\pi f_1 t + \phi_1) \cdot A_2 \cos(2\pi f_2 t + \phi_2) \\ &= \frac{A_1 A_2}{2} \cos(2\pi [f_1 - f_2] t + \phi_1 - \phi_2) + \frac{A_1 A_2}{2} \cos(2\pi [f_1 + f_2] t + \phi_1 + \phi_2) \end{aligned} \quad (1)$$

It also shows what happens when you multiply two sinusoids having the same frequency

$$\begin{aligned} v(t) &= A_1 \cos(2\pi f t + \phi_1) \cdot A_2 \cos(2\pi f t + \phi_2) \\ &= \frac{A_1 A_2}{2} \cos(\phi_1 - \phi_2) + \frac{A_1 A_2}{2} \cos(4\pi f t + \phi_1 + \phi_2) \end{aligned} \quad (2)$$

Addition of two sinusoids (CEP 2.3.3)

The addition of two sinusoids having different frequencies cannot be simplified any further

$$v(t) = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2)$$

The addition of two sinusoids having the same frequency can be simplified further

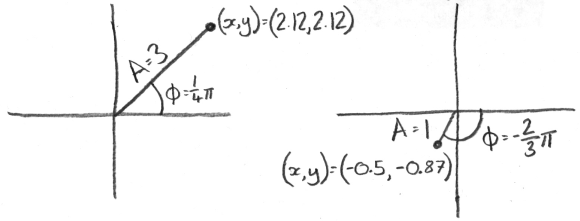
$$\begin{aligned} v(t) &= A_1 \cos(2\pi f t + \phi_1) + A_2 \cos(2\pi f t + \phi_2) \\ &= A \cos(2\pi f t + \phi) \end{aligned}$$

The result is a single sinusoid having the same frequency, but a new amplitude and phase.

Phasors (CEP 2.3.3.2)

e.g. $v(t) = 3\cos(2\pi f t + \frac{1}{4}\pi) + \cos(2\pi f t - \frac{2}{3}\pi)$

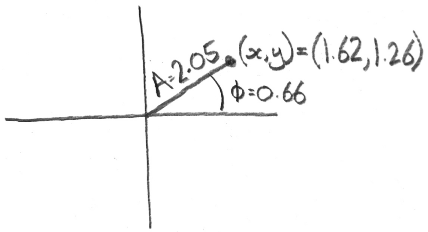
- Represent each sinusoid with a phasor



- Determine the coordinates (x, y) of each phasor using trigonometry

  $x = A\cos(\phi) \quad y = A\sin(\phi)$

- Add the coordinates to get the resultant phasor



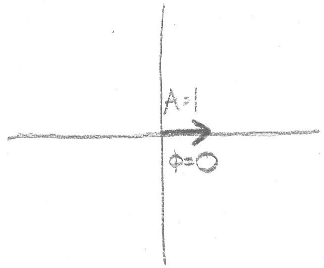
- Determine the amplitude A and phase ϕ of the phasor

$$A = \sqrt{x^2 + y^2} \qquad \phi = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x \geq 0 \\ \tan^{-1}\left(\frac{y}{x}\right) - \pi & \text{if } x < 0 \end{cases}$$

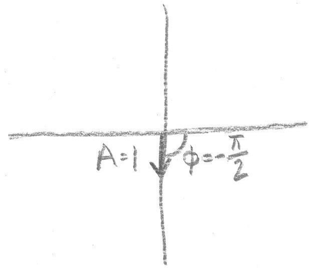
so $v(t) = 2.05\cos(2\pi f t + 0.66)$

Phasors continued

$\cos(2\pi f t)$



$\sin(2\pi f t) = \cos(2\pi f t - \frac{\pi}{2})$

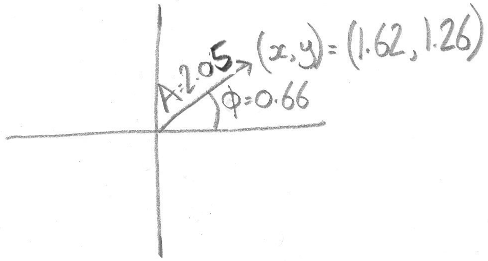


Any phasor can be thought of as a sum of a cos and a sin

$$A\cos(2\pi f t + \phi) = x\cos(2\pi f t) - y\sin(2\pi f t)$$

where $x = A\cos(\phi)$ and $y = A\sin(\phi)$ as before

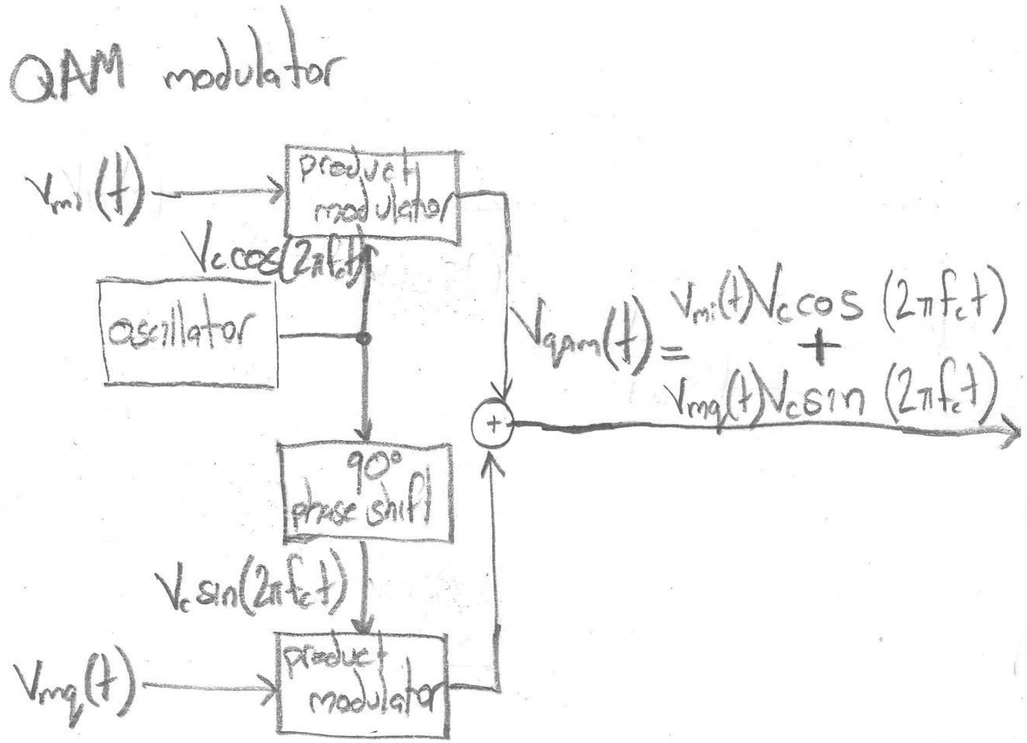
eg $v(t) = 2.05\cos(2\pi f t + 0.66)$ from before



$$v(t) = 1.62\cos(2\pi f t) - 1.26\sin(2\pi f t)$$

QAM revisited

- QAM uses an in-phase carrier $V_c \cos(2\pi f_c t)$ and a quadrature-phase carrier $V_c \sin(2\pi f_c t)$ having the same frequency.



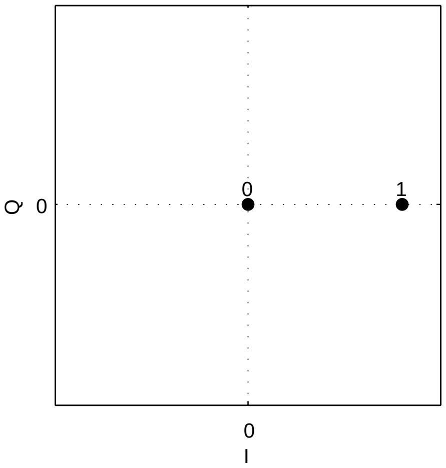
- The x coordinate of a phasor is equivalent to the in-phase signal $v_{mi}(t)$ of QAM.
- The y coordinate of a phasor is equivalent to the quadrature-phase signal $-v_{mq}(t)$.

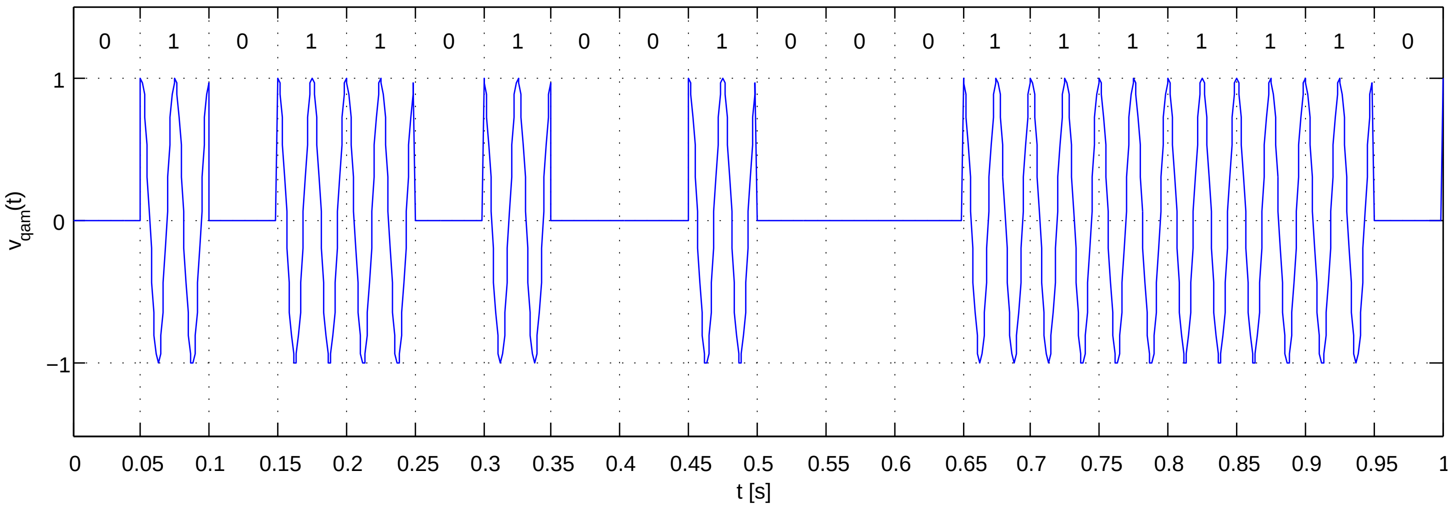
Constellation diagrams (CEP 7.9.2, 7.9.3 and 7.9.5)

- At the start of this lecture, we quickly looked at ASK, PSK and FSK digital carrier modulation schemes.
- For ASK and PSK, phasors can be drawn in a constellation diagram to show how the amplitude and phase of the carrier is modulated in order to signal different combinations of bit values.
- Each of the $M = 2^k$ constellation points is labelled with a different combination of k bits.
- The constellation points can be positioned and labelled in many different ways. Here are some examples...

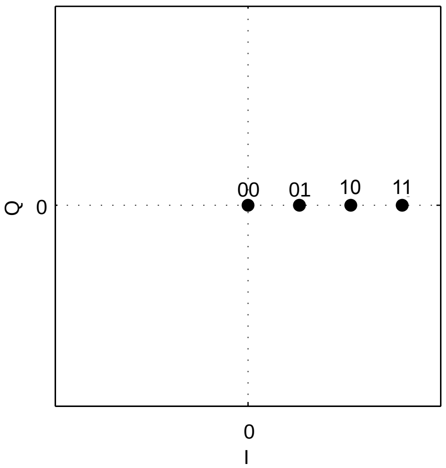
Constellation diagrams (CEP 7.9.2)

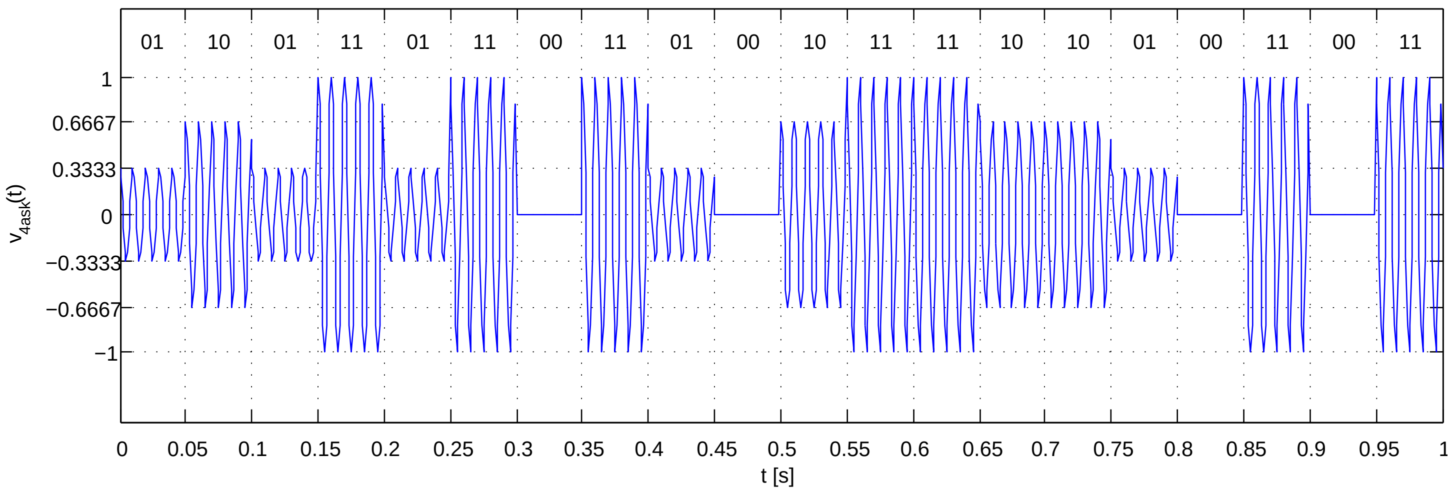
- On-Off Keying (OOK)





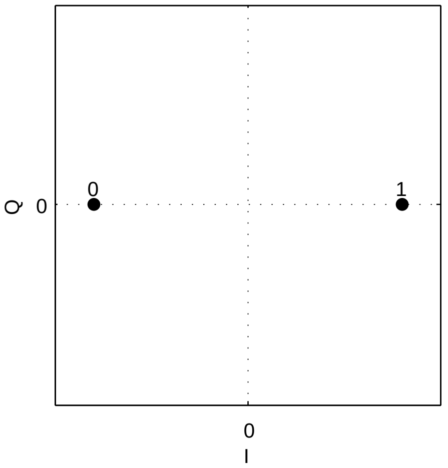
- $M = 4$-ary Amplitude Shift Keying (4ASK)

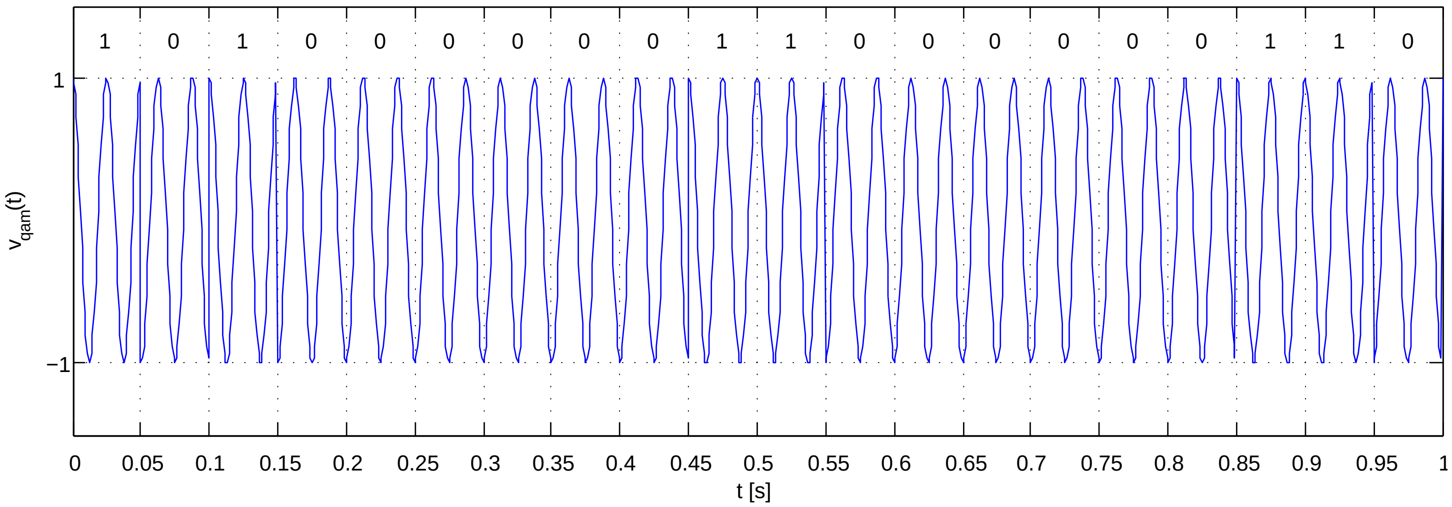




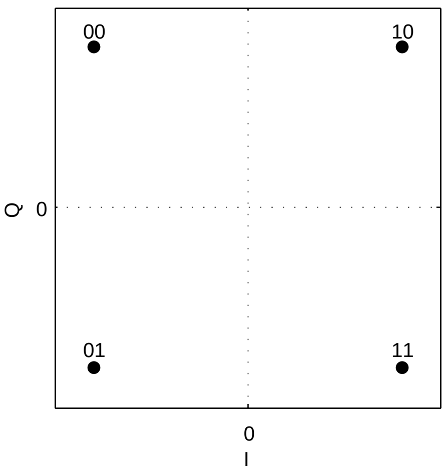
Constellation diagrams (CEP 7.9.3)

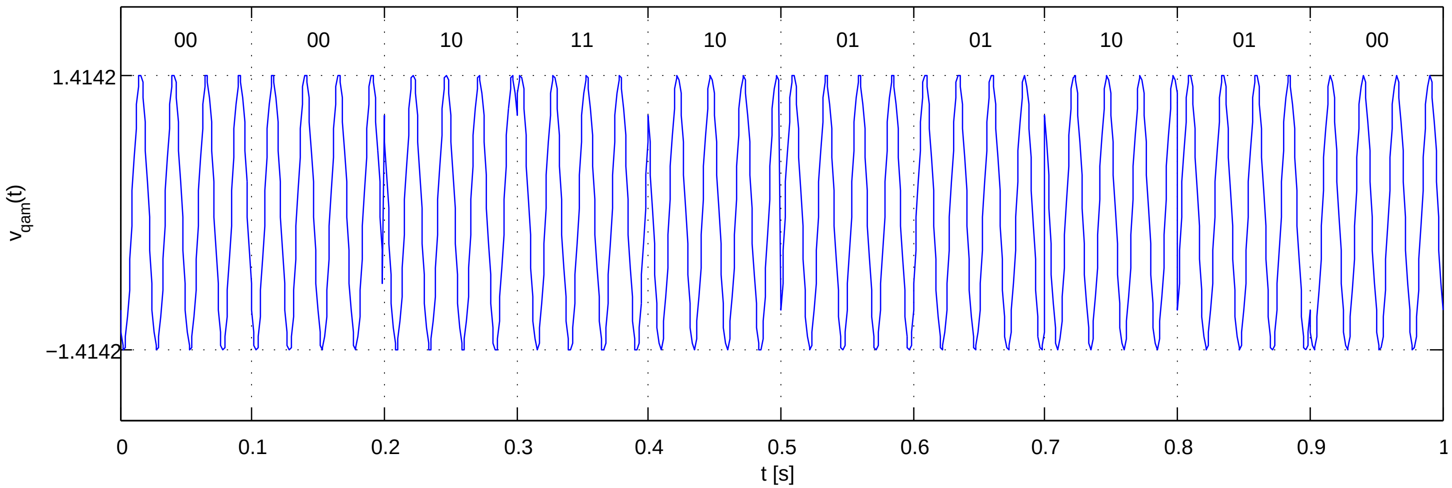
- Binary ($M = 2$) Phase Shift Keying (BPSK)





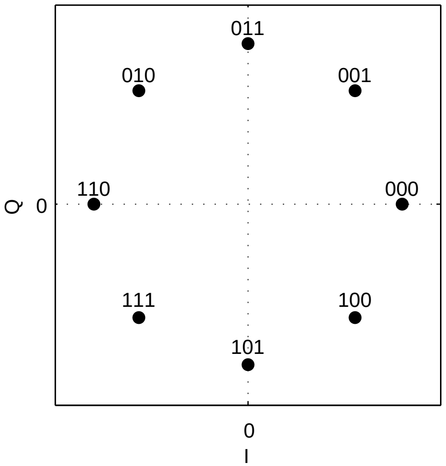
- Quarternary ($M = 4$) Phase Shift Keying (QPSK)

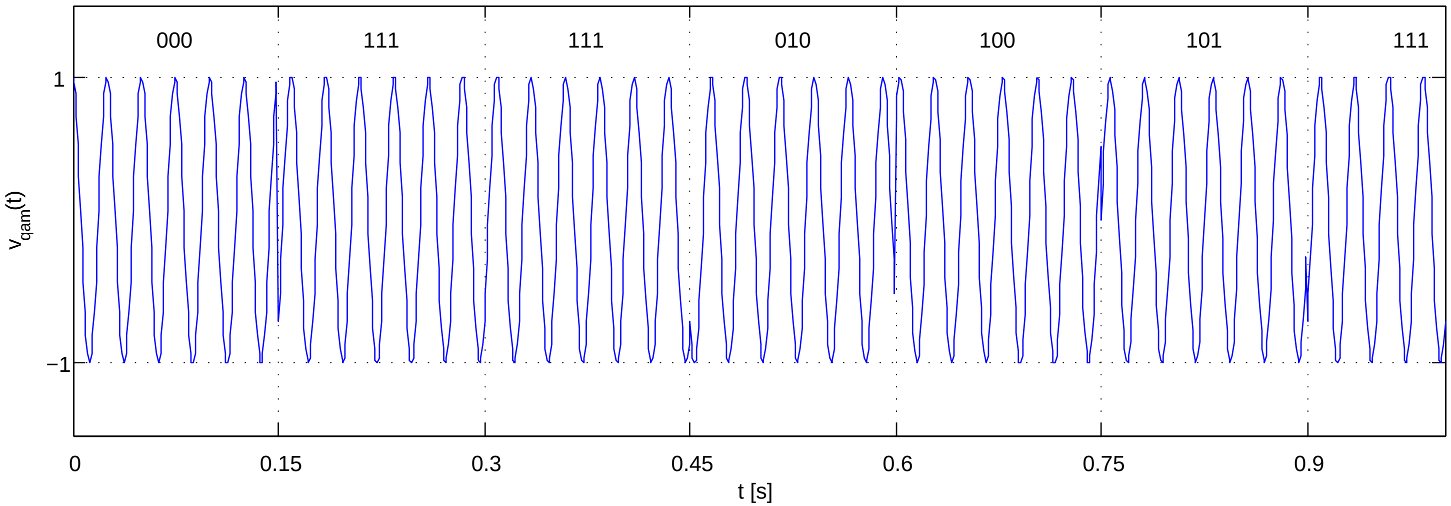




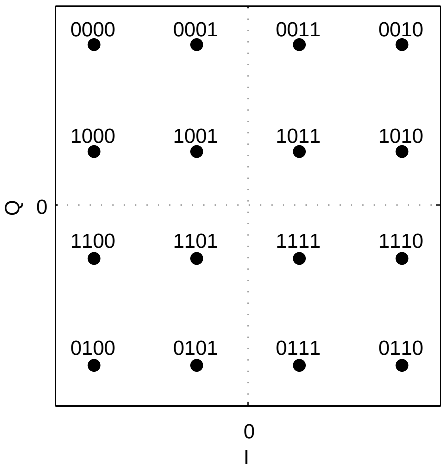
Constellation diagrams (CEP 7.9.3 and 7.9.5)

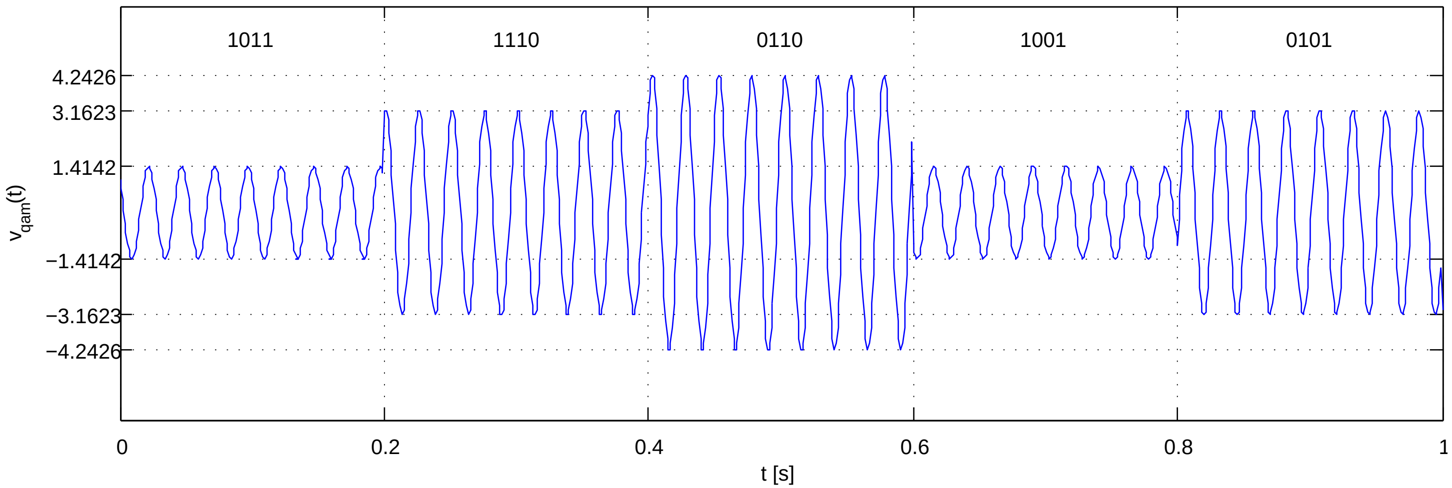
- $M = 8$-ary Phase Shift Keying (8PSK)





- $M = 16$-ary Quadrature Amplitude Modulation (16QAM)





Exercise

1. Sketch OOK, BFSK and BPSK digital carrier modulations of the bit sequence 0111110000111011 using a bit rate of 80 bit/s, remembering to annotate the time axes.

2. Draw phasor diagrams for the sinusoids $3\cos(200\pi t + \pi/4)$ and $-2\sin(200\pi t)$.

3. Express the sum of these sinusoids as a single sinusoid and draw the corresponding phasor diagram.

4. Express the new sinusoid as the sum of in-phase and quadrature-phase sinusoids.