Question 1.

(a) Let the TBox T with the following axioms:

$$\text{UndergraduateModule} \sqsubseteq \exists\, \text{taughtBy.Lecturer}$$

$$\text{Lecturer} \sqsubseteq \text{Academic}$$

$$\exists\, \text{taughtBy.Academic} \sqsubseteq \text{Module}$$

$$\text{UndergraduateModule} \sqcap \text{Lecturer} \sqsubseteq \bot$$

Let Abox $\mathcal{A}$ containing the following assertions:

$$\text{UndergraduateModule(Comp1215)} \quad \text{Lecturer(George)}$$
$$\text{Academic(Nick)}$$

Under the closed world assumption, give the certain answer (TRUE or FALSE) to each of the following boolean queries for $(\mathcal{T}, \mathcal{A})$:

(i) Lecturer(George)

(ii) Lecturer(Nick)

(iii) $\text{Lecturer(Comp1215)}$

(iv) Module(George)

(v) Module(Nick)

(vi) Module(Comp1215)

(vii) Academic(George)

(viii) Academic(Nick)

(ix) $\text{Academic(Comp1215)}$

[10 marks]

(b) Translate the following four sentences into description logic axioms. Use only concept names Student, Module, Course, the role name registeredFor, and the individual name Comp6215.

(i) Every student is registered for at least 3 modules [3 marks]

(ii) Students are registered only for modules or courses [4 marks]

(iii) Courses are not registered for anything [4 marks]

(iv) Everyone registered for Comp6215 is a student [4 marks]

(c) Consider the following interpretation consisting of a set Δ (the domain of discourse) and an interpretation function ext:

$$\begin{aligned} \Delta &= \{ a, b, c, d, e, f, g \} \\ ext(A) &= \{ c, d \} \\ ext(B) &= \{ d, e, f \} \\ ext(R) &= \{ \langle a, a \rangle, \langle a, c \rangle, \langle c, c \rangle, \langle d, c \rangle, \langle e, g \rangle, \langle f, g \rangle \} \\ ext(S) &= \{ \langle a, d \rangle, \langle d, a \rangle, \langle e, f \rangle, \langle f, e \rangle \} \end{aligned}$$

Give the values of ext for the following class expressions:
(e.g. $ext(A \sqcap B) = \{ d \}$):

(i) $ext(A \sqcap \neg B)$ [2 marks]

(ii) $ext(A \sqcap \exists R.\{c\})$ [2 marks]

(iii) $ext(\forall R.\neg A)$ [2 marks]

(iv) $ext(\exists S.\forall R.A)$ [2 marks]

TURN OVER

Indicative Solution for Question 1.

(a) (i) $\mathbf{Lecturer(George)}$ **Yes**

(ii) $\mathbf{Lecturer(Nick)}$ **No**

(iii) $\mathbf{Lecturer(Comp1215)}$ **No**

(iv) $\mathbf{Module(George)}$ **No**

(v) $\mathbf{Module(Nick)}$ **No**

(vi) $\mathbf{Module(Comp1215)}$ **Yes**

(vii) $\mathbf{Academic(George)}$ **Yes**

(viii) $\mathbf{Academic(Nick)}$ **Yes**

(ix) $\mathbf{Academic(Comp1215)}$ **No**

(1 mark each)

(b) (i) $\mathbf{Student} \sqsubseteq (\geq 3\mathbf{registeredFor.Module})$ **(3 marks)**

(ii) $\mathbf{Student} \sqsubseteq \forall\mathbf{registeredFor.}(\mathbf{Module} \sqcup \mathbf{Course})$ **(4 marks)**

(iii) $\mathbf{Course} \sqsubseteq \forall\mathbf{registeredFor.} \bot$ **(4 marks)**

(iv) $\exists\mathbf{registeredFor.}\{\mathrm{Comp6215}\} \sqsubseteq \mathbf{Student}$ **(4 marks)**

(c) (i) $ext(A \sqcap \neg B) = \{\ c\ \}$

(ii) $ext(A \sqcap \exists R.\{c\}) = \{\ c, d\ \}$

(iii) $ext(\forall R.\neg A) = \{\ b, e, f, g\ \}$

(iv) $ext(\exists S.\forall R.A) = \{a\}$

(2 marks each)

Question 5.

(a) Description logics (DL) may be considered to be subsets of first-order predicate logic (FOPL). Consider the following DL axiom:

$$\text{Happy} \sqcap \text{Child} \sqsubseteq \exists\, \text{loves}^{-}.\text{Person}$$

(i) Show how the DL axiom may be expressed in natural language. [2 marks]

(ii) Show how the DL axiom may be expressed in FOPL. [6 marks]

(b) Consider the following interpretation consisting of a set Δ (the domain of discourse) and an interpretation function ext:

$$\begin{aligned} \Delta &= \{\, a, b, c, d, e, f, g \,\} \\ ext(A) &= \{\, c, d \,\} \\ ext(B) &= \{\, d, e, f \,\} \\ ext(R) &= \{\, \langle a, a\rangle, \langle a, c\rangle, \langle c, c\rangle, \langle d, c\rangle, \langle e, g\rangle, \langle f, g\rangle \,\} \\ ext(S) &= \{\, \langle a, d\rangle, \langle d, a\rangle, \langle e, f\rangle, \langle f, e\rangle \,\} \end{aligned}$$

Give the values of ext for the following class expressions:
(e.g. $ext(A \sqcap B) = \{\, d \,\}$):

(i) $ext(A \sqcap \neg B)$ [2 marks]

(ii) $ext(A \sqcap \exists R.\{c\})$ [2 marks]

(iii) $ext(\forall R.\neg A)$ [2 marks]

(iv) $ext(\exists S.\forall R.A)$ [2 marks]

(v) $ext(S^{+})$ [2 marks]

TURN OVER

(c) Represent the following statements in Description Logics:

(i) Every small child likes dinosaurs. [3 marks]

(ii) Every carnivorous dinosaur only eats meat. [3 marks]

(iii) A triceratops is a ceratopsid with three horns. [3 marks]

(iv) No triceratops eats meat. [3 marks]

(v) All birds are descended from dinosaurs. [3 marks]

Indicative Solution for Question 5.

(a) **(i)** **"Every happy child is loved by some person" (or similar)**

(2 marks)

(ii)

$$\forall x.\phi_{\mathbf{Happy}}(x) \wedge \phi_{\mathbf{Child}}(x) \Rightarrow \exists y.\phi_{\mathbf{loves}}(y,x) \wedge \phi_{\mathbf{Person}}(y)$$

(6 marks)

(b) **(i)** $ext(A \sqcup \neg B) = \{\ c\ \}$

(ii) $ext(A \sqcap \exists R.\{c\}) = \{\ c, d\ \}$

(iii) $ext(\forall R.\neg A) = \{\ b, e, f, g\ \}$

(iv) $ext(\exists S.\forall R.A) = \{a\}$

(v) $ext(S^+) = \{\ \langle a,a\rangle, \langle a,d\rangle, \langle d,a\rangle, \langle d,d\rangle, \langle e,e\rangle, \langle e,f\rangle, \langle f,e\rangle, \langle f,f\rangle,\ \}$

(2 marks each)

(c) **(i)** $Child \sqcap Small \sqsubseteq \exists likes.Dinosaur$

(ii) $Dinosaur \sqcap Carnivore \sqsubseteq \forall eats.Meat$

(iii) $Triceratops \equiv Ceratopsid \sqcap = 3 hasHorn$

(iv) $Triceratops \sqcap \exists eats.Meat \equiv \bot$

(v) $Bird \sqsubseteq \exists descendedFrom.Dinosaur$

(3 marks each)

END OF PAPER

Question 5.

(a) Description logics (DL) may be considered to be subsets of first-order predicate logic (FOPL). Consider the following DL axiom:

$$\text{Dog} \sqcap \text{Young} \sqsubseteq \exists \text{ learns.Trick}$$

(i) Show how the DL axiom may be expressed in natural language. [2 marks]

(ii) Show how the DL axiom may be expressed in FOPL. [6 marks]

(b) Consider the following interpretation consisting of a set Δ (the domain of discourse) and an interpretation function ext:

$$\begin{aligned} \Delta &= \{\ a, b, c, d, e, f, g\ \} \\ ext(A) &= \{\ b, c, d\ \} \\ ext(B) &= \{\ d, e, f\ \} \\ ext(R) &= \{\ \langle b, a\rangle, \langle b, c\rangle, \langle b, e\rangle, \langle e, f\rangle, \langle f, b\rangle, \langle f, g\rangle\ \} \\ ext(S) &= \{\ \langle b, c\rangle, \langle b, d\rangle, \langle d, d\rangle, \langle e, d\rangle, \langle f, g\rangle\ \} \end{aligned}$$

Give the values of ext for the following class expressions:
(e.g. $ext(A \sqcap B) = \{\ d\ \}$):

(i) $ext(A \sqcup \neg B)$ [2 marks]

(ii) $ext(B \sqcap \exists S.\{d\})$ [2 marks]

(iii) $ext(\forall R.B)$ [2 marks]

(iv) $ext(A \sqcap \exists R.\exists R.A)$ [2 marks]

(v) $ext(R \circ S)$ [2 marks]

(c) Represent the following statements in Description Logics:

(i) Every animal is either tame or wild. [3 marks]

(ii) A pet is a tame animal. [3 marks]

(iii) Every cat likes both fish and cream. [3 marks]

(iv) No cats like baths. [3 marks]

(v) A cat lover is a person who only has pets that are cats. [3 marks]

TURN OVER

Indicative Solution for Question 5.

(a) **(i)** **"Every young dog learns tricks" (or similar)**

(2 marks)

(ii)

$$\forall x.\phi_{\mathbf{Dog}}(x) \wedge \phi_{\mathbf{Young}}(x) \Rightarrow \exists y.\phi_{\mathbf{learns}}(x,y) \wedge \phi_{\mathbf{Trick}}(y)$$

(6 marks)

(b) **(i)** $ext(A \sqcup \neg B) = \{\ a, b, c, d, g\ \}$

(ii) $ext(B \sqcap \exists S.\{d\}) = \{\ d, e, f\ \}$

(iii) $ext(\forall R.B) = \{\ a, c, d, e, g\ \}$

(iv) $ext(A \sqcap \exists R.\exists R.A) = \{\}$

(v) $ext(R \circ S) = \{\ \langle b, d\rangle, \langle e, d\rangle, \langle f, d\rangle\ \}$

(2 marks each)

(c) **(i)** $Animal \sqsubseteq Tame \sqcup Wild$

(ii) $Pet \equiv Animal \sqcap Tame$

(iii) $Cat \sqsubseteq \exists likes.(Fish \sqcup Cream)$

(iv) $Cat \sqcap \exists likes.Bath \equiv \bot$

(v) $CatLover \equiv Person \sqcap \forall hasPet.Cat$

(3 marks each)

END OF PAPER

Question 5.

(a) Description logics (DL) may be considered to be fragments of first-order predicate logic (FOPL). Consider the following DL axiom:

$$\text{Martian} \sqcap \text{Young} \sqsubseteq \exists\ \text{likes}.\text{Rocket}$$

(i) Show how the DL axiom may be expressed in natural language. [2 marks]

(ii) Show how the DL axiom may be expressed in FOPL. [6 marks]

(b) Consider the following interpretation consisting of a set Δ (the domain of discourse) and an interpretation function ext:

$$\begin{aligned} \Delta &= \{\ a, b, c, d, e, f, g, h\ \} \\ ext(A) &= \{\ c, d, f, g\ \} \\ ext(B) &= \{\ b, c, g, h\ \} \\ ext(R) &= \{\ \langle a, b\rangle, \langle b, c\rangle, \langle c, g\rangle, \langle g, b\rangle\ \} \\ ext(S) &= \{\ \langle e, f\rangle, \langle f, d\rangle, \langle f, g\rangle, \langle g, h\rangle\ \} \end{aligned}$$

Give the values of ext for the following class expressions:
(e.g. $ext(A \sqcap B) = \{\ c, g\ \}$):

(i) $ext(A \sqcap \neg B)$ [2 marks]

(ii) $ext(A \sqcap \exists S.A)$ [2 marks]

(iii) $ext(\forall S.(\exists R.A))$ [2 marks]

(iv) $ext(\exists S^{-}.A)$ [2 marks]

(v) $ext(R^{+})$ [2 marks]

TURN OVER

(c) Represent the following statements in Description Logics:

(i) A Martian is an alien that comes from Mars. [3 marks]

(ii) A Thoat is a Martian animal with eight legs. [3 marks]

(iii) Every Martian has either red or green skin. [3 marks]

(iv) No green Martians can read. [3 marks]

(v) Every two-headed Martian rides a thoat. [3 marks]

Indicative Solution for Question 5.

(a) **(i)** **"Every young Martian likes rockets" (or similar)**

(2 marks)

(ii)

$$\forall x.\phi_{\mathbf{Martian}}(x) \wedge \phi_{\mathbf{Young}}(x) \Rightarrow \exists y.\phi_{\mathbf{likes}}(x,y) \wedge \phi_{\mathbf{Rocket}}(y)$$

(6 marks)

(b) **(i)** $ext(A \sqcap \neg B) = \{\ d, f\ \}$

(ii) $ext(A \sqcap \exists S.A) = \{\ f\ \}$

(iii) $ext(\forall S.(\exists S.A)) = \{\ a, b, c, d, e, h\ \}$

(iv) $ext(\exists S^{-}.A) = \{\ d, g, h\ \}$

(v) $ext(R^{+}) = \{\ \langle a,b\rangle, \langle a,c\rangle, \langle a,g\rangle, \langle b,c\rangle, \langle b,g\rangle, \langle b,b\rangle, \langle c,g\rangle, \langle c,b\rangle, \langle c,c\rangle, \langle g,b\rangle, \langle g,c\rangle, \langle g,g\rangle\ \}$

(2 marks each)

(c) **(i)** $Martian \equiv Alien \sqcap \exists comesFrom.\{\ mars\ \}$

(ii) $Thoat \equiv Animal \sqcap \exists comesFrom.\{\ mars\ \} \sqcap = 8\ hasLegs$

(iii) $Martian \sqsubseteq \exists hasSkin.(Red \sqcup Green)$

(iv) $\exists hasSkin.Green \sqcap Martian \sqcap Reader \equiv \bot$

(v) $=\ 2\ hasHead \sqcap Martian \sqsubseteq \exists rides.Thoat$

(3 marks each)

END OF PAPER

Question 5.

(a) Description logics (DL) may be considered to be fragments of first-order predicate logic (FOPL). Consider the following DL axiom:

$$\text{Child} \sqcap \exists\ \text{owns.Toy} \sqsubseteq \text{Happy}$$

(i) Show how the DL axiom may be expressed in natural language. [2 marks]

(ii) Show how the DL axiom may be expressed in FOPL. [6 marks]

(b) Consider the following interpretation consisting of a set Δ (the domain of discourse) and an interpretation function ext:

$$\begin{aligned} \Delta &= \{\ a, b, c, d, e, f, g\ \} \\ ext(A) &= \{\ b, c, d\ \} \\ ext(B) &= \{\ d, e, f\ \} \\ ext(C) &= \{\ f\ \} \\ ext(R) &= \{\ \langle b, b\rangle, \langle b, c\rangle, \langle f, c\rangle, \langle f, g\rangle, \langle d, b\rangle\ \} \\ ext(S) &= \{\ \langle d, e\rangle, \langle e, d\rangle, \langle f, f\rangle, \langle f, g\rangle\ \} \end{aligned}$$

Give the values of ext for the following class expressions:
(e.g. $ext(A \sqcap B) = \{\ d\ \}$):

(i) $ext(\neg B)$ [2 marks]

(ii) $ext(B \sqcap \neg C)$ [2 marks]

(iii) $ext(\forall S.B)$ [2 marks]

(iv) $ext(\exists S.(\exists R.A))$ [2 marks]

(v) $ext(R^{-})$ [2 marks]

(c) Represent the following statements in Description Logics:

(i) A Martian is an alien who comes from the planet Mars. [3 marks]

(ii) No Martian can climb trees. [3 marks]

(iii) Every young Martian likes rockets. [3 marks]

(iv) Every happy Martian has a child who pilots a rocket. [3 marks]

(v) Every Martian rocket pilot has visited Earth. [3 marks]

TURN OVER

Indicative Solution for Question 5.

(a) **(i)** **"Every child who owns a toy is happy" (or similar)**

(2 marks)

(ii)

$$\forall x.\phi_{\mathbf{Child}}(x) \wedge \exists y.\phi_{\mathbf{owns}}(x,y) \wedge \phi_{\mathbf{Toy}}(x) \Rightarrow \phi_{\mathbf{Happy}}(x)$$

(6 marks)

(b) **(i)** $ext(\neg B) = \{\ a, b, c, g\ \}$

(ii) $ext(B \sqcap \neg C) = \{\ d, e\ \}$

(iii) $ext(\forall S.B) = \{\ a, b, c, d, e, g\ \}$

(iv) $ext(\exists S.(\exists R.A)) = \{\ e, f\ \}$

(v) $ext(R^{-}) = \{\ \langle b, b\rangle, \langle c, b\rangle, \langle c, f\rangle, \langle g, f\rangle, \langle b, d\rangle\ \}$

(2 marks each)

(c) **(i)** $Martian \equiv Alien \sqcap \exists comesFrom.\{\ mars\ \}$

(ii) $Martian \sqcap \exists climbs.Tree \equiv \bot$

(iii) $Martian \sqcap Young \sqsubseteq \exists likes.Rocket$

(iv) $Martian \sqcap Happy \sqsubseteq \exists hasChild.(\exists pilots.Rocket)$

(v) $Martian \sqcap \exists pilots.Rocket \sqsubseteq \exists visited.\{\ earth\ \}$

(3 marks each)

END OF PAPER

Question 5.

(a) Description logics (DL) may be considered to be fragments of first-order predicate logic (FOPL). Consider the following DL axiom:

$$\text{Cheeseburger} \equiv \text{Burger} \sqcap \exists\ \text{hasTopping}.\text{Cheese}$$

(i) Show how the DL axiom may be expressed in natural language. [2 marks]

(ii) Show how the DL axiom may be expressed in FOPL. [6 marks]

(b) Consider the following interpretation consisting of a set Δ (the domain of discourse) and an interpretation function ext:

$$\begin{aligned} \Delta &= \{\ a, b, c, d, e, f, g\ \} \\ ext(A) &= \{\ b, c, d\ \} \\ ext(B) &= \{\ d, e\ \} \\ ext(R) &= \{\ \langle b, c\rangle, \langle c, b\rangle, \langle d, e\rangle, \langle e, d\rangle, \langle f, g\rangle, \langle g, f\rangle\ \} \\ ext(S) &= \{\ \langle a, b\rangle, \langle a, c\rangle, \langle c, d\rangle, \langle d, b\rangle, \langle d, g\rangle, \langle f, e\rangle\ \} \end{aligned}$$

Give the values of ext for the following class expressions:
(e.g. $ext(A \sqcap B) = \{\ d\ \}$):

(i) $ext(\neg A)$ [2 marks]

(ii) $ext(A \sqcap \neg B)$ [2 marks]

(iii) $ext(\geq 2\ R)$ [2 marks]

(iv) $ext(\forall R.A)$ [2 marks]

(v) $ext(\exists S.(\exists R.B))$ [2 marks]

TURN OVER

(c) Represent the following statements in Description Logics:

(i) Every child who owns a toy is happy [3 marks]

(ii) A children's meal is a combo meal with a toy [3 marks]

(iii) Every combo meal contains one burger, one drink and at least one side dish [3 marks]

(iv) Every child who only eats things that aren't vegetables is unhealthy [3 marks]

(v) No unhealthy child is happy [3 marks]

Indicative Solution for Question 5.

(a) **(i)** **"A cheeseburger is a burger with a cheese topping" (or similar)**

(2 marks)

(ii)

$$\forall x.\phi_{\mathbf{Cheeseburger}}(x) \implies \phi_{\mathbf{Burger}}(x) \wedge \exists y.\phi_{\mathbf{hasTopping}}(x,y) \wedge \phi_{\mathbf{Cheese}}(y)$$

(6 marks)

(b) **(i)** $ext(\neg A) = \{a, e, f, g\}$

(ii) $ext(A \sqcap \neg B) = \{b, c\}$

(iii) $ext(\geq 2\ S) = \{a, d\}$

(iv) $ext(\forall R.A) = \{a, b, c, e\}$

(v) $ext(\exists S.(\exists R.B)) = \{c, f\}$

(2 marks each)

(c) **(i)** $Child \sqcap \exists owns.Toy \sqsubseteq Happy$

(ii) $ChildMeal \equiv ComboMeal \sqcap \exists contains.Toy$

(iii) $ComboMeal \sqsubseteq = 1\ contains.Burger \sqcap = 1\ contains.Drink \sqcap \exists contains.Side$

(iv) $Child \sqcap \forall eats.\neg Vegetable \sqsubseteq Unhealthy$

(v) $Child \sqcap Unhealthy \sqcap Happy \equiv \bot$

(3 marks each)

END OF PAPER