

# Topology of Networks and Data

Ruben Sanchez-Garcia  
Mathematical Sciences

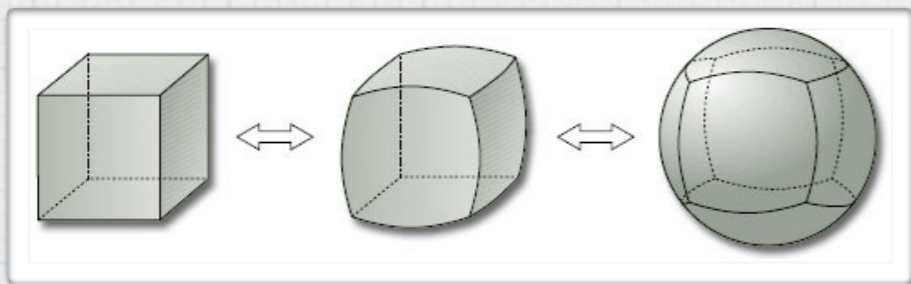
WebScience MSc Guest Lecture  
8 December 2016

# 1-slide summary

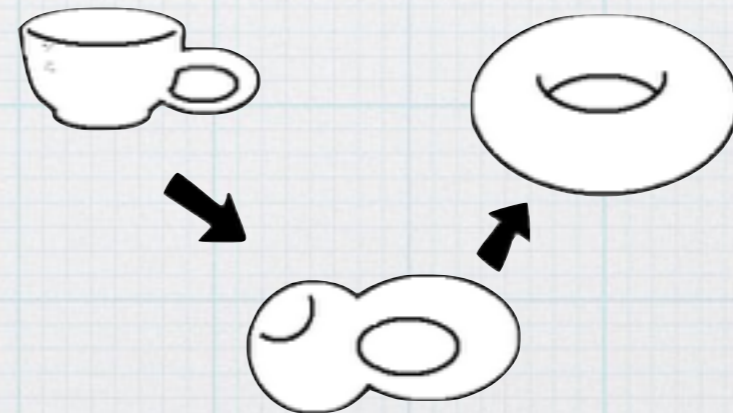
- \* I use TOPOLOGY to study complex systems, and data
- \* I use networks, and their higher-dimensional analogues, complexes, to represent 'shape'
- \* Data: mostly interested in unsupervised methods, particularly clustering, and ranking

# Topology

- \* Mathematical study of 'shapes'
- \* Similar to Geometry, but objects can be continuously deformed



<http://www.renyi.mta.hu/~szilard/topology/CUBE=SPHERE.jpeg>



<http://atomsandvoid.files.wordpress.com/2011/02/topology1.png?w=570>

# Seven bridges of Königsberg

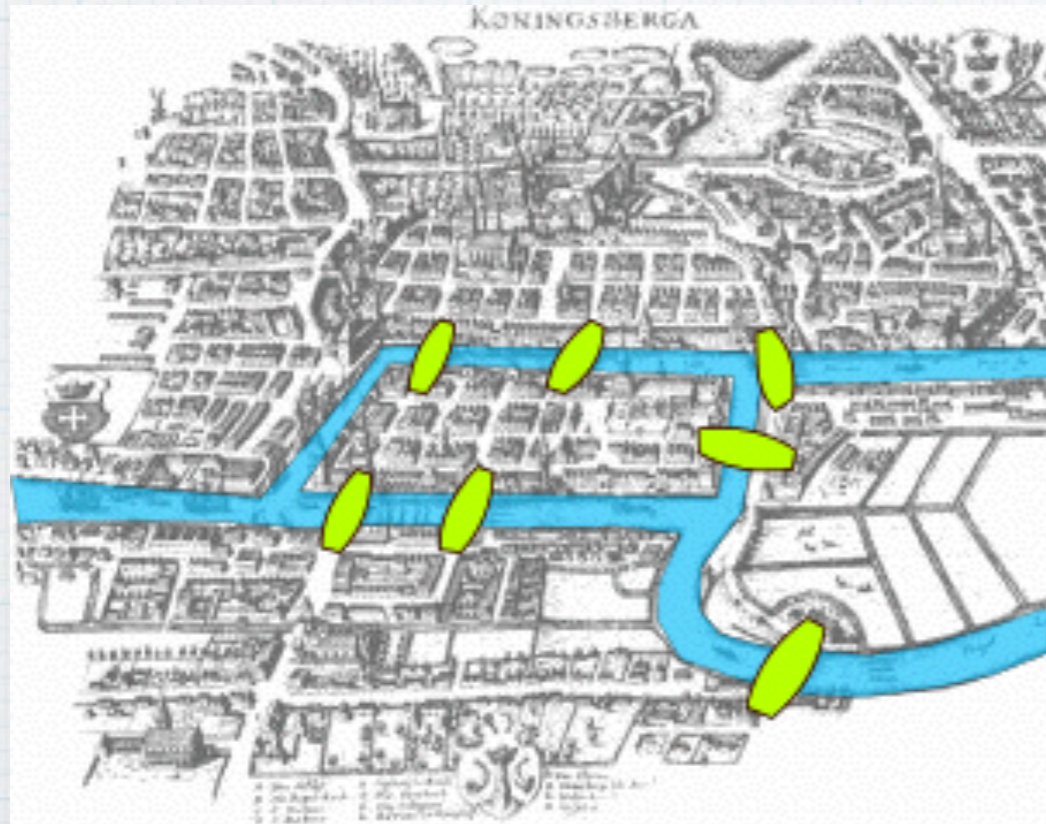
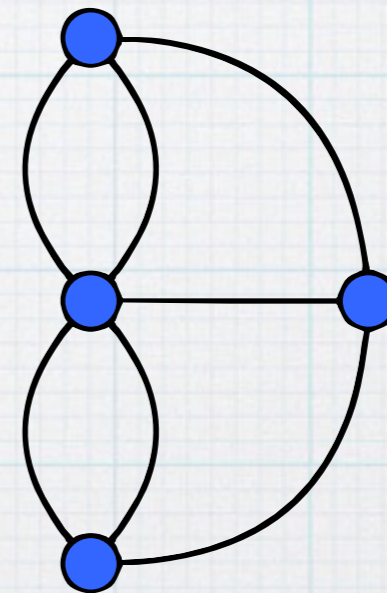


Image by Bogdan Giușcă (Wikipedia)



Euler (1735)

# Discrete structures in Topology

\* **Complexes: Discrete representations of continuous shapes**

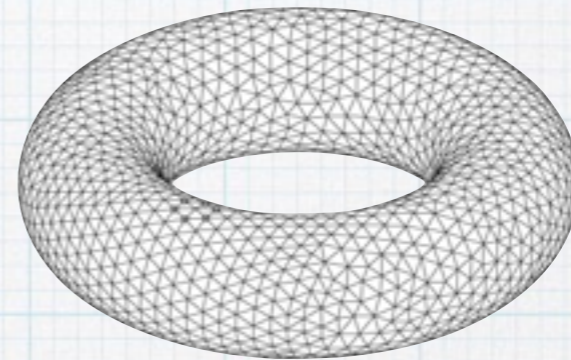


Image by Ag2gaeh (Wikipedia)

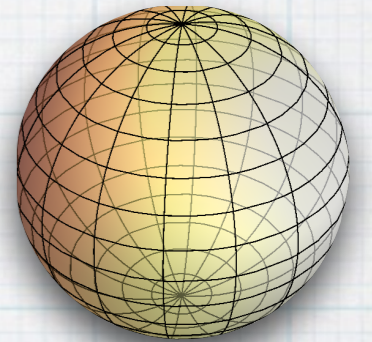
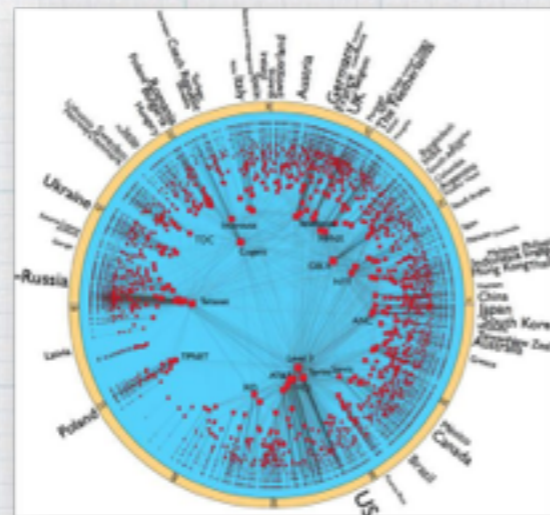


Image: Mathematica

\* **Example: Networks**



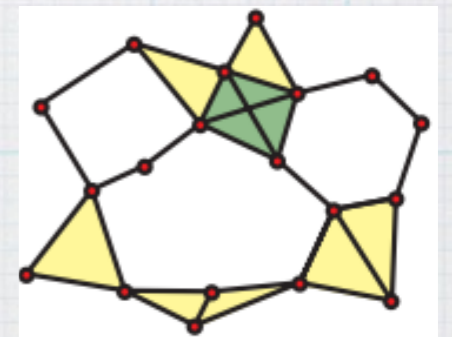
biological



technological



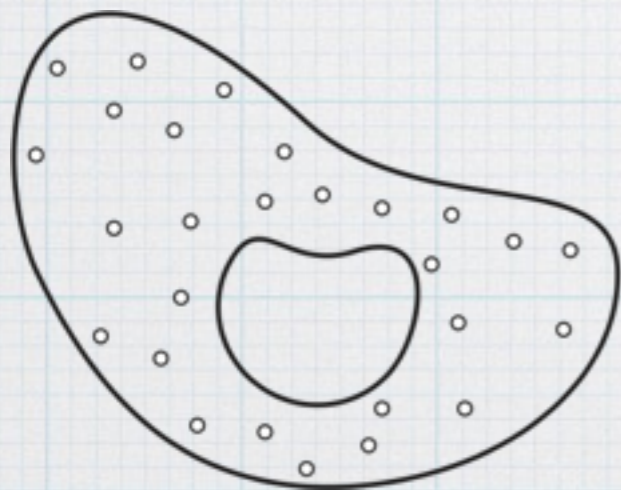
socio-technological



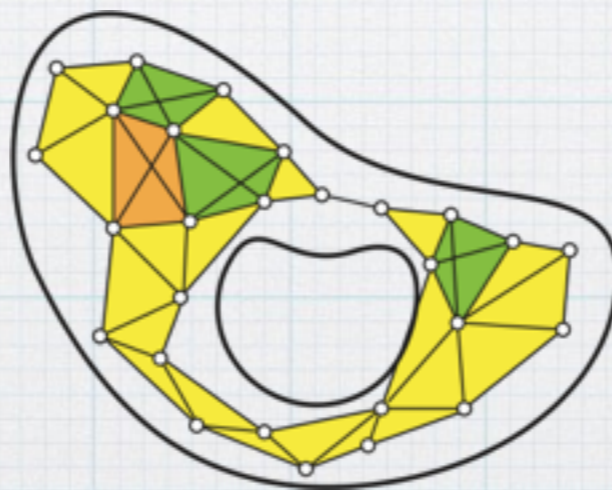
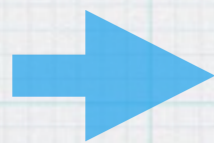
abstract complex

# From Data to Complexes

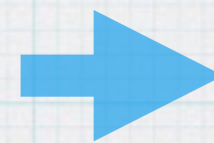
- \* **DATA** = data points  $x_i$  with pairwise distances (or similarities)  $s_{ij}$
- \* **Example:** point clouds in high-dimensional space



point cloud



data complex



**TOPOLOGICAL  
FEATURES**

# Topological Features

\* Example: Euler characteristics of a complex

$$\chi = \#nodes - \#links + \#faces$$

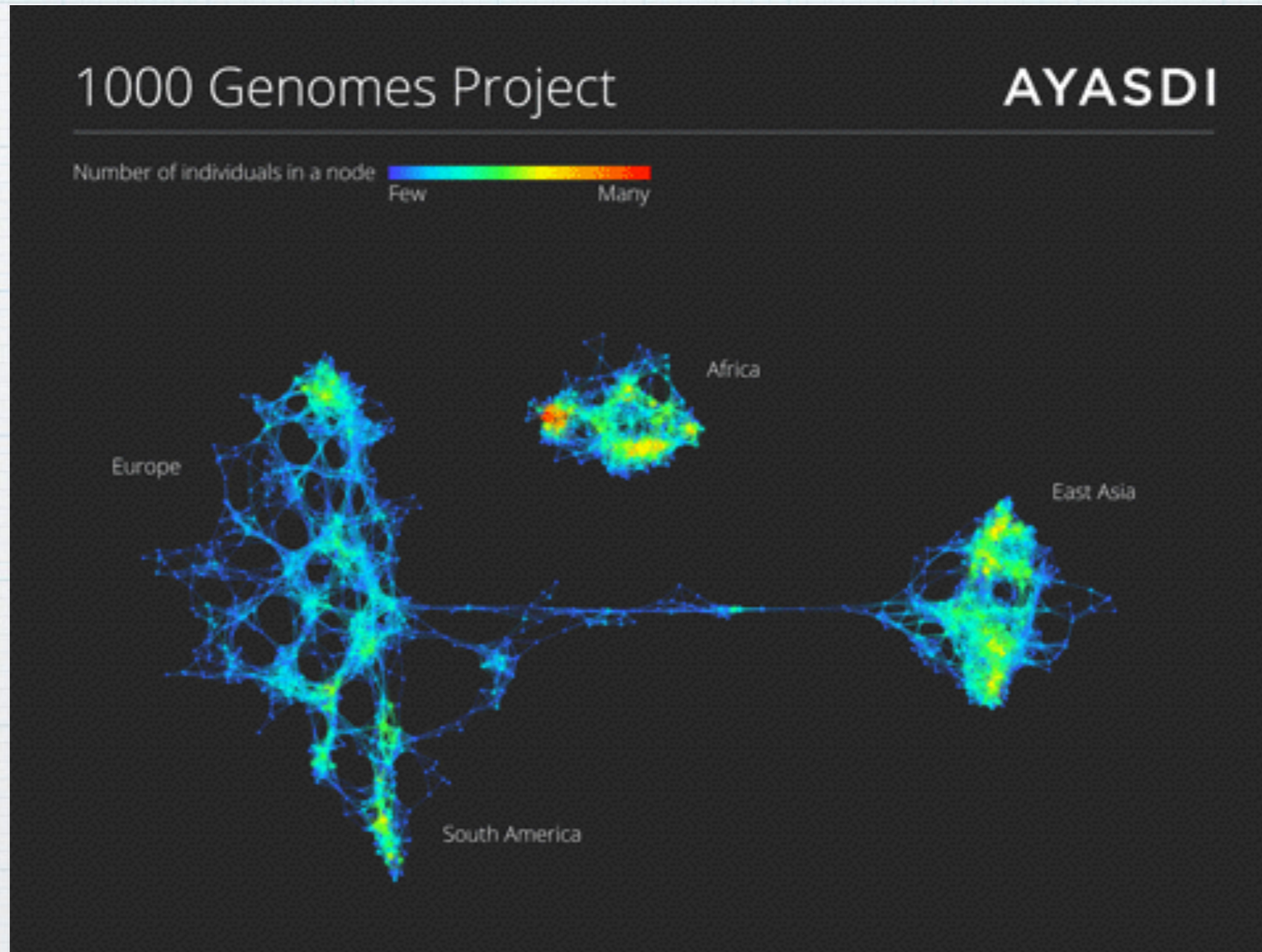
$$\chi(\text{Sphere}) = 2$$

$$\chi(\text{Figure-eight}) = -1$$

$$\chi(\text{Torus}) = 0$$

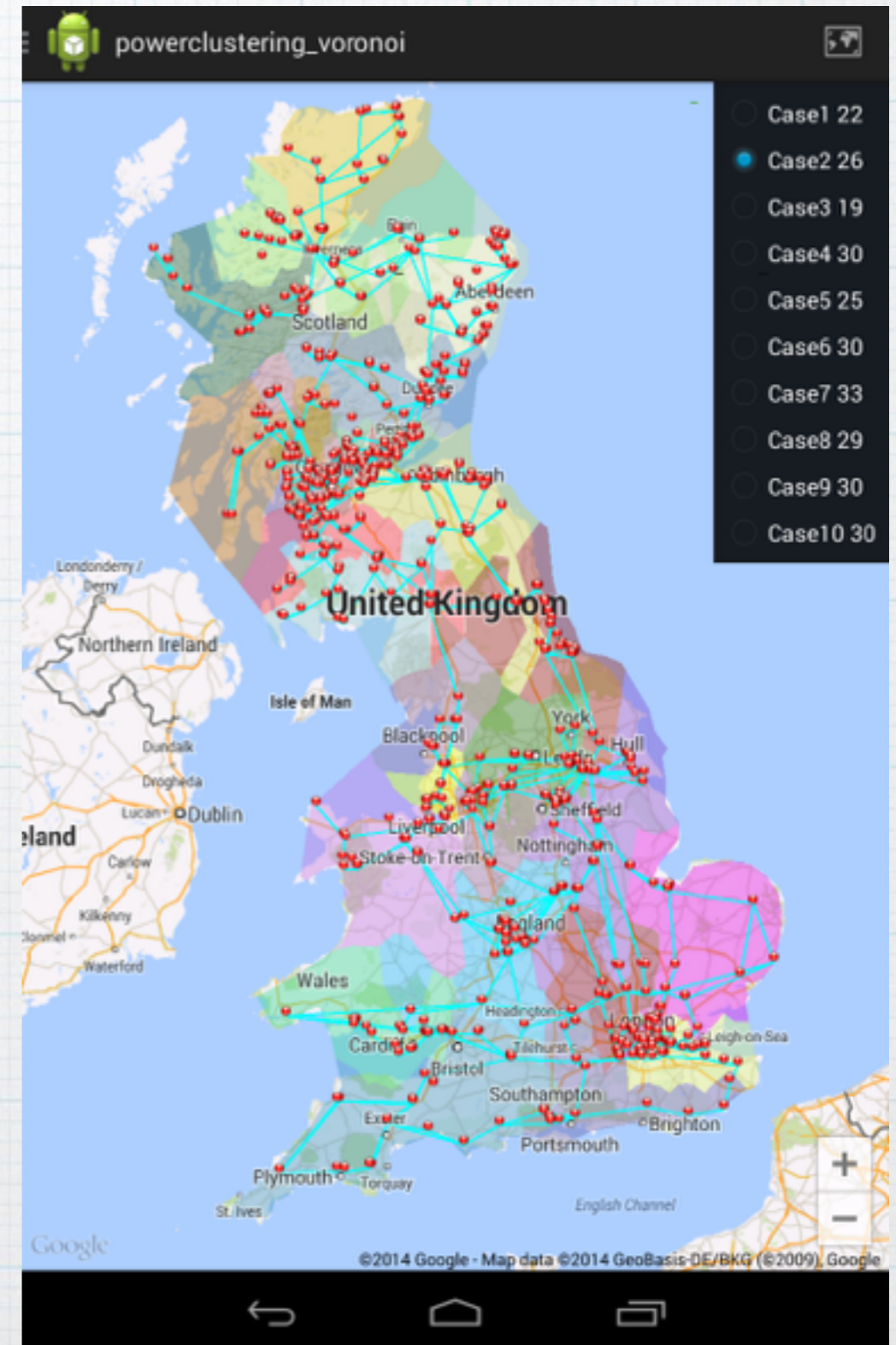
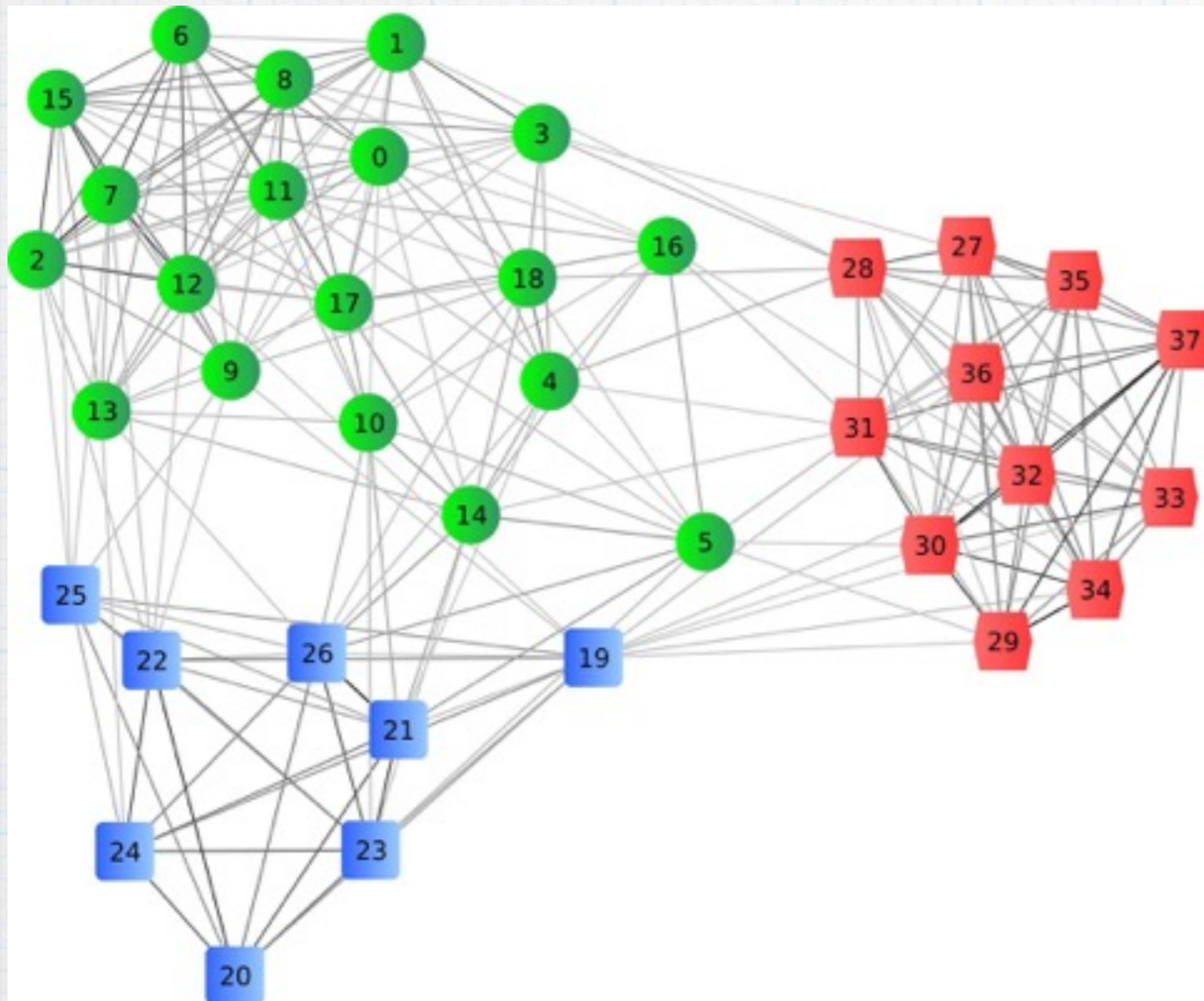
Etc.

# Topological Data Analysis





# Clustering/Community detection



\* Spectral clustering: network eigenvectors reveal clustering structure

# Topological Ranking

- \* Current WebScience student Conrad D'Souza
- \* Ranking is a key activity that permeates the Web
- \* Challenges: missing data, inconsistencies
- \* Topological ranking (HodgeRank): global ranking & explains residual error
- \* Case study: horse racing data

# 1-slide summary (again!)

- \* I use **TOPOLOGY** to study complex systems, and data
- \* I use networks, and their higher-dimensional analogues, complexes, to represent 'shape'
- \* Data: mostly interested in unsupervised methods, particularly clustering, and ranking

# Want to know more?

[1] G. Carlsson, Topology and data, *Bulletin of the American Mathematical Society*, 2009.

[2] JavaPlex: Persistent homology and topological data analysis library <http://javaplex.github.io/javaplex/>

[3] Mapper in Python (Daniel Müllner) <http://danifold.net/mapper/index.html>

[4] Ayasdi <http://www.ayasdi.com/>

[5] I. J. Good, The Philosophy of Exploratory Data Analysis, *Philosophy of Science*, 1983.

[6] M. Nicolau, A. J. Levine, and G. Carlsson, Topology based data analysis identifies a subgroup of breast cancers with a unique mutational profile and excellent survival, *PNAS*, 2011.

[7] J. M. Chan, G. Carlsson, and R. Rabadan, Topology of viral evolution, *PNAS*, 2013.

[8] Sayan Mukherjee: <https://stat.duke.edu/~sayan/Publications.html>

## or talk to me

[R.Sanchez-Garcia@soton.ac.uk](mailto:R.Sanchez-Garcia@soton.ac.uk)

Office 8023 in Building 54

Thank you