

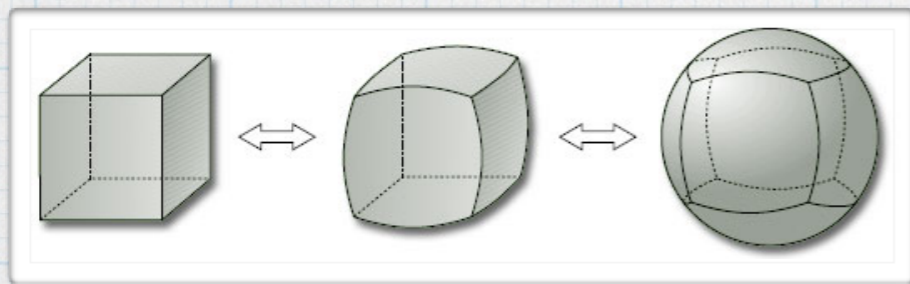
Topology of Networks and Data

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Mathematical Sciences

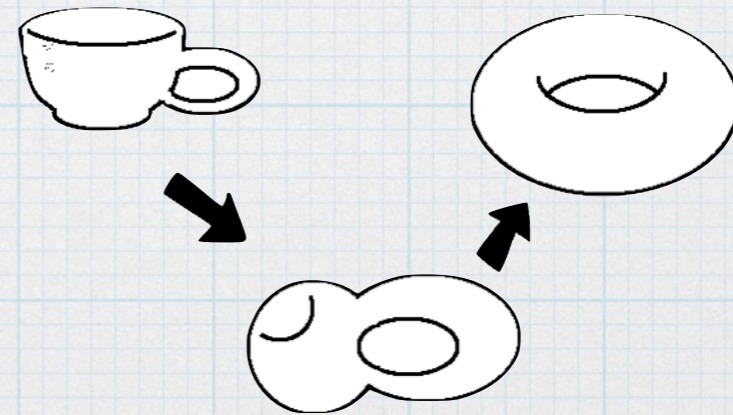
WebScience MSc Guest Lecture
18 November 2015

Topology

- * Mathematical study of 'shapes'
- * Similar to Geometry, but objects can be continuously deformed



<http://www.renyi.mta.hu/~szilard/topology/CUBE=SPHERE.jpeg>



<http://atomsandvoid.files.wordpress.com/2011/02/topology1.png?w=570>

Seven bridges of Königsberg

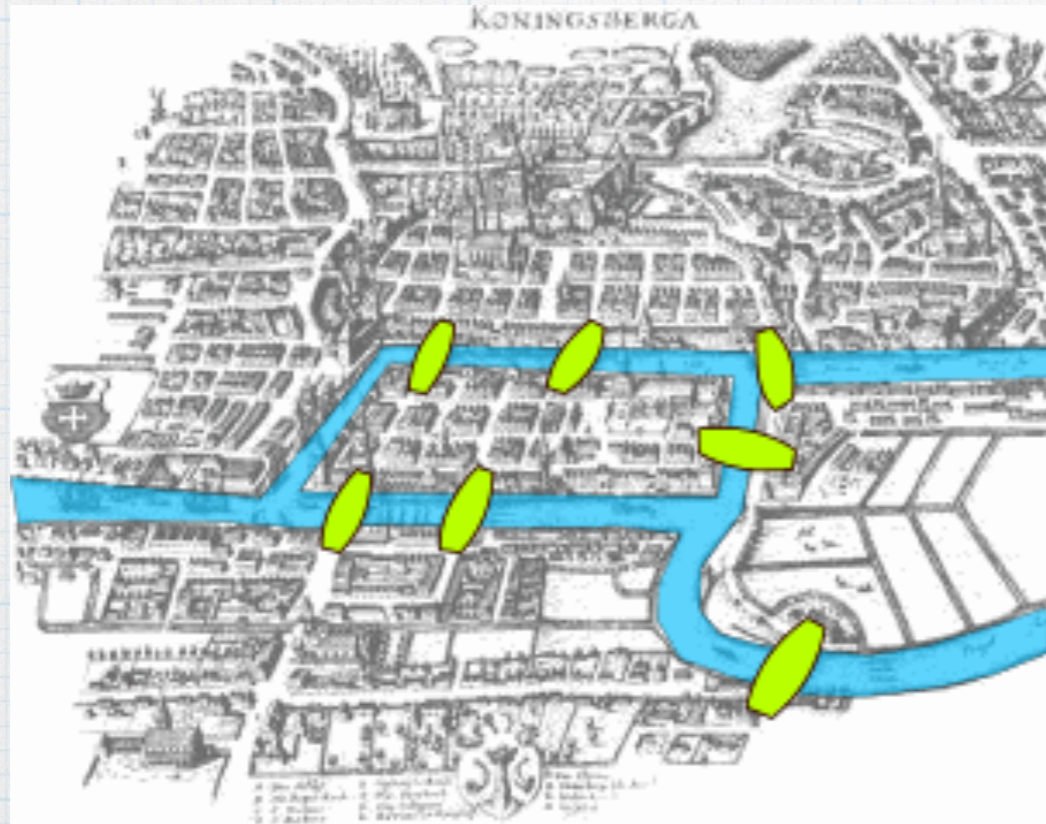
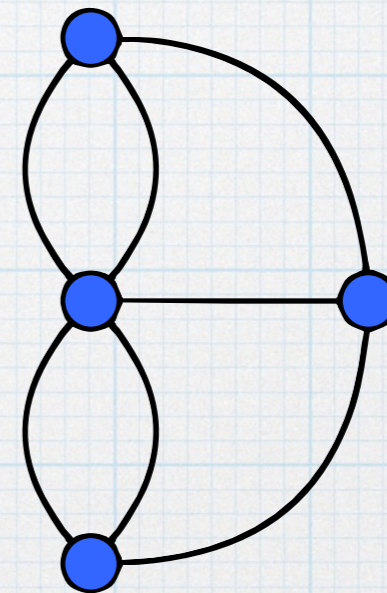


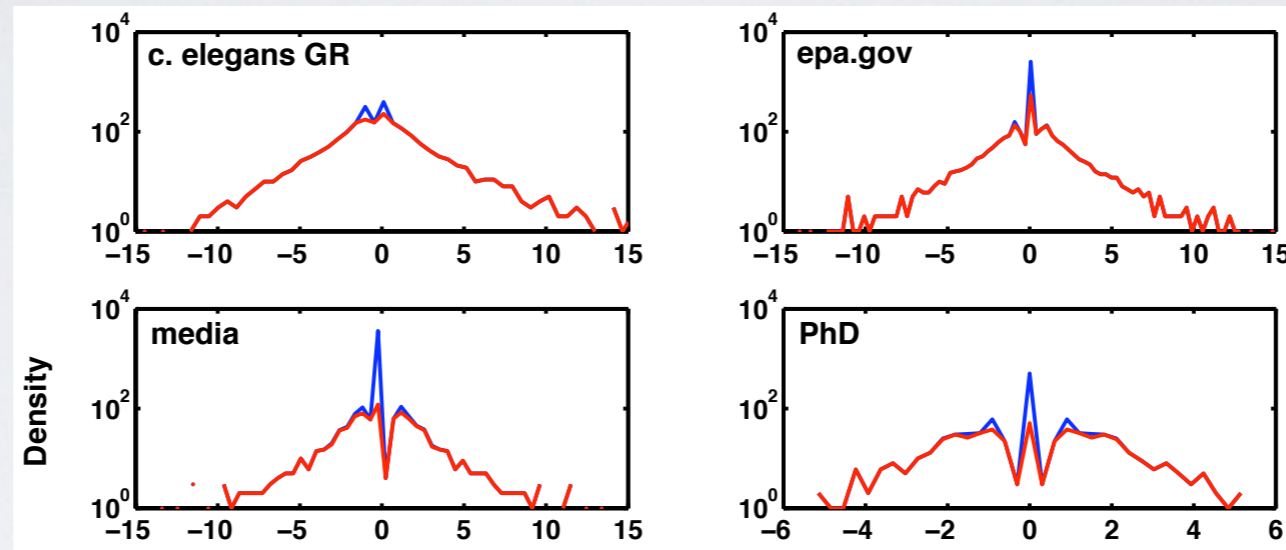
Image by Bogdan Giușcă (Wikipedia)



Euler (1735)

Symmetric spectrum

- We studied how symmetries (automorphisms) affect network spectrum
- Symmetries give rise to high-multiplicity eigenvalues (peaks in spectral density)



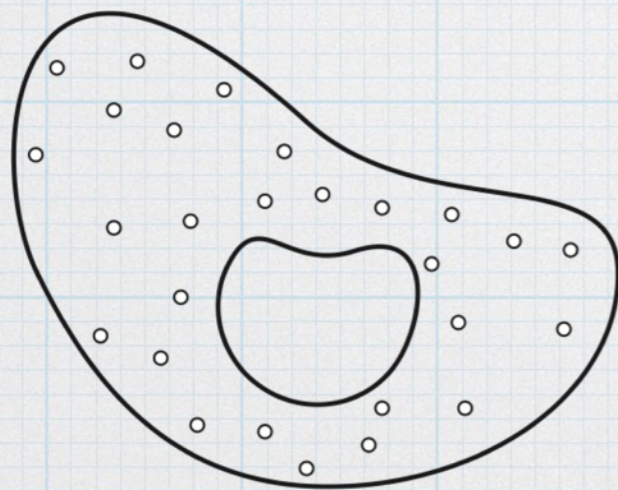
- The network spectrum is the union of the *redundant* spectrum of the symmetric motifs, and the spectrum of the quotient network
- The redundant spectrum of the *basic* symmetric motifs is very constrained
e.g. $\text{RSpec}_1 = \{-1, 0\}$ $\text{RSpec}_2 = \{-2, -\varphi, -1, 0, \varphi - 1, 1\}$
 $\text{RSpec}_3 = \{-3, -2, -1, 0, 1, \pm\sqrt{2}, \pm\sqrt{3}, -1 \pm \sqrt{2}, -1 \pm \sqrt{3}, \dots\}$

MacArthur, Sanchez-Garcia, Anderson *Symmetry in Complex Networks* **Discrete Appl. Math.** (2008)

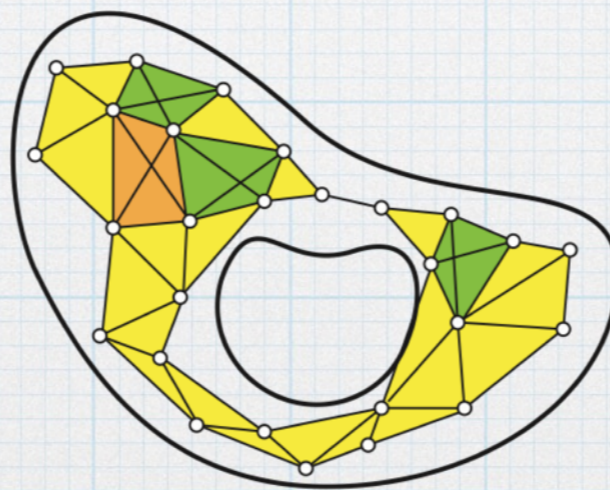
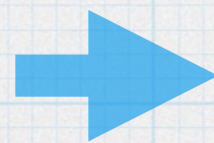
MacArthur, Sanchez-Garcia *Spectral characteristics of network redundancy* **Phys. Rev. E** (2009)

From Data to Complexes

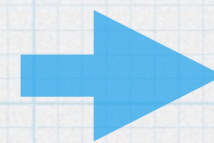
- * DATA = entities x_i with pairwise distances or similarities s_{ij}
- * Example: point clouds in high-dimensional space



point cloud



data complex



TOPOLOGICAL
FEATURES

Topological Features

* Example: Euler characteristics of a complex

$$\chi = \#nodes - \#links + \#faces$$

$$\chi(\text{Sphere}) = 2$$

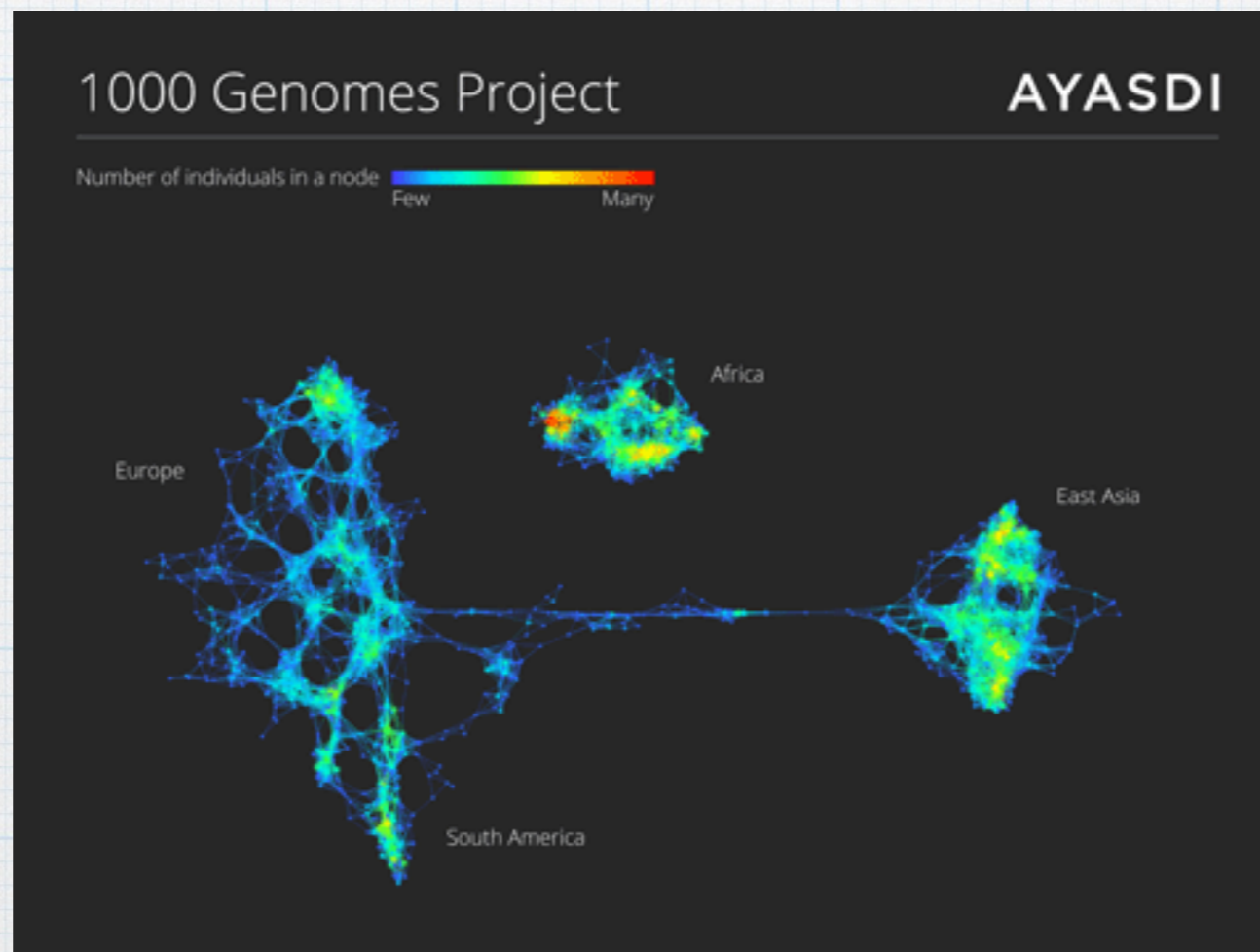
$$\chi(\text{Figure-eight}) = -1$$

$$\chi(\text{Torus}) = 0$$

Etc.

Topology as a visualisation tool

- * Visualisation: Exploratory data analysis



AYASDI

Topological Ranking

- * Project with current WebScience MSc/PhD student Conrad D'Souza
- * Ranking is a key activity that permeates the Web
- * Challenges: sparse data, transitivity, etc.
- * Topological rank: global ranking & explains residual error
- * Case study: horse racing data

Want to know more?

[1] G. Carlsson, Topology and data, *Bulletin of the American Mathematical Society*, 2009.

[2] JavaPlex: Persistent homology and topological data analysis library <http://javaplex.github.io/javaplex/>

[3] Mapper in Python (Daniel Müllner) <http://danifold.net/mapper/index.html>

[4] Ayasdi <http://www.ayasdi.com/>

[5] I. J. Good, The Philosophy of Exploratory Data Analysis, *Philosophy of Science*, 1983.

[6] M. Nicolau, A. J. Levine, and G. Carlsson, Topology based data analysis identifies a subgroup of breast cancers with a unique mutational profile and excellent survival, *PNAS*, 2011.

[7] J. M. Chan, G. Carlsson, and R. Rabadan, Topology of viral evolution, *PNAS*, 2013.

[8] Sayan Mukherjee: <https://stat.duke.edu/~sayan/Publications.html>

or talk to me

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Thank you