

Maths 3018/6111 - Numerical Methods

Worksheet 1

Theory

1. Write down the 1, 2 and ∞ vector norms of

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 6 \\ -3 \\ 1 \end{pmatrix}.$$

2. Find the 1 and ∞ matrix norms of

$$A_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -3 & 2 \\ 3 & 6 \end{pmatrix}$$

3. Find the condition numbers of the above matrices. What does this suggest about the numerical behaviour of an algorithm that used such a matrix?
4. Explain the difference between direct and indirect methods for solving linear systems. Give an example of when the latter may be more useful.

Coding

1. Enter each of the matrices above into Matlab. For each matrix, work out the transpose and inverse using in-built Matlab commands.
2. Check your calculation of the vector and matrix norms, and the condition numbers.
3. Write a function that takes a matrix, computes its condition number, and reports whether the matrix is suitably well-conditioned (the choice of criteria is up to you).
4. [3018 only] Write a bisection method to find the root of

$$f(x) = \tan x - e^{-x}, \quad x \in [0, 1].$$