Maths 3018/6111 - Numerical Methods

Worksheet 3

Theory

1. Apply Simpson's rule to compute
$$\int_0^{\pi/2} \cos(x)\, dx$$
using 3 points (so $h = \pi/4$) and 5 points (so $h = \pi/8$).

2. Apply Richardson extrapolation to the result above; does the answer improve?

3. State the rate of convergence of the trapezoidal rule and Simpson's rule, and sketch (or explain in words) the proof.

4. Explain in words adaptive and Gaussian quadrature, in particular the aims of each and the times when one or the other is more useful.

5. [3018 only] Show how the speed of convergence of a nonlinear root finding method depends on the derivatives of the map $g(x)$ near the fixed point s.

6. [3018 only] Use Newton's method to find the root in $[0, 1]$ of
$$f(x) = \sin(x) - e^x + 0.9 + x.$$
Start from $x_0 = 1/2$ and retain 3 significant figures. Take 3 steps.

Coding

1. Write a single function that, depending on an options argument, computes the integral of an input function $f(x)$ between the input arguments a, b, using either

   (a) Simpson's rule, 3 points

   (b) Trapezoidal rule, 3 points

   (c) Gaussian Quadrature, 3 nodes.

   Test your code on
$$\int_0^1 \sin^2(\pi x)\, dx = \tfrac{1}{2},$$
$$\int_0^1 e^{-x} \sinh(x)\, dx \simeq 0.283833821$$
$$\int_0^1 \frac{1}{\sqrt{x}}\, dx = 2.$$
   Note that there are good reasons for some of the methods to fail on the final test!

2. [3018 only] Implement the secant method to find the root of
$$f(x) = \tan x - e^{-x}, \quad x \in [0, 1].$$