Maths 3018/6111 - Numerical Methods

Worksheet 4

Theory

1. Convert the ODE
$$y''' + xy'' + 3y' + y = e^{-x}$$
into a first order system of ODEs.

2. Show by Taylor expansion that the backwards differencing estimate of $f'(x)$,
$$f'(x) \simeq \frac{f(x) - f(x-h)}{h}$$
is first order accurate.

3. Use Taylor expansion to derive a symmetric or central difference estimate of $f^{(4)}(x)$ on a grid with spacing h.

4. State the convergence rate of Euler's method and the Euler predictor-corrector method.

5. Explain when multistage methods such as Runge-Kutta methods are useful.

6. [3018 only] Explain the power method for finding the largest eigenvalue of a matrix. In particular, explain why it is simpler to find the absolute value, and how to find the phase information.

Coding

1. Apply Euler's method to the ODE
$$y' + 2y = 2 - e^{-4x}, \quad y(0) = 1.$$
Find the value of $y(1)$ (analytic answer is $1 - (e^{-2} - e^{-4})/2$) and see how your method converges with resolution.

2. Apply the standard RK4 method to the above system, again checking that it converges with resolution.

3. [3018 only] Write a code using the power method and inverse power method to compute the largest and smallest eigenvalues of an arbitrary matrix. Apply it to a random $n = 3$ matrix, checking that the correct answer is found. How does the number of iterations required for convergence to a given level vary with the size of the matrix?